

Bihar Engineering University, Patna
End Semester Examination – 2023
Semester-IV

Course: B.Tech.

Code: 100404

Subject: Discrete Mathematics

Time: 03 Hours

Full Marks: 70

Instructions:-

- (i) The marks are indicated in the right-hand margin.
- (ii) There are NINE questions in this paper.
- (iii) Attempt FIVE questions in all.
- (iv) Question No. 1 is compulsory.

Q.1 Write the answer of the following (Any seven question only):

[2 x 7 = 14]

- a) Let A be the set odd positive integers less than 10. Then cardinality of A, $|A|$ is
 - (i) 5
 - (ii) 9
 - (iii) 6
 - (iv) 4
- b) If m is the number of objects (pigeons) and n is the number of boxes (pigeonholes), then the function is both one – to – one and onto if
 - (i) $m < n$ (ii) $m = n$ (iii) $m > n$ (iv) none of these
- c) If $A \times B = B \times A$, (Where A and B are general matrices) then
 - (i) $A = d$ (ii) $A = B'$ (iii) $B = A$, (iv) $A' = B$
- d) A partial ordered relation is transitive, reflexive and
 - (i) Anti symmetric (ii) bi symmetric (iii) anti reflexive (iv) asymmetric
- e) If B is a Boolean Algebra, then which of the following is true
 - i. B is a finite but complemented lattice
 - ii. B is a finite, complemented and distributive lattice
 - iii. B is a finite, Distributive but not complemented lattice
 - iv. B is not distributive lattice
- f) $P \rightarrow q$ is logically equivalent to
 - a) $\sim q \rightarrow p$ b) $\sim P \rightarrow q$
 - c) $\sim P \wedge q$.d) $\sim p \vee q$
- g) if $f(x) = \cos x$ and $g(x) = x^3$ then $(f \circ g)(x)$ is
 - (i) $(\cos x)^3$ (ii) $\cos 3x$ (iii) $x^{(\cos x)^3}$ (iv) $\cos x^3$
- h) The number of distinguishable permutations of the letters in the word BANANA are
 - (i) 60 (ii) 36, (iii) 20, (iv) 10
- i) Which of the following pair is not congruent modulo 7?
 - (i) 10, 24
 - (ii) 25, 56
 - (iii) -31, -15
 - (iv) -64, -15
- j) Let $N = \{1, 2, 3, \dots\}$ be ordered by divisibility, which of the following subset is totally ordered
 - (i) (2, 6, 24) (ii) (3, 5, 15) (iii) (2, 9, 16), (iv) (4, 15, 30)
- Q2. a) Let $A = B = \{x \mid 1 \leq x \leq 1\}$ for each of the following functions state where it is injective, surjective or bijective
 - i) $g(x) = \sin \pi x$ [7]
 - ii) $b(x) = \frac{2x}{3}$
- b) Let $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$ find (i) $f \circ g$ (ii) $f \circ g \circ h$ [7]
- Q3. a) find the power set of each of these sets [7]
 - i) $\{a, b\}$ ii) $\{\varnothing, \{\varnothing\}\}$
- b.) Use Cantor's diagonal argument to prove that set F of all functions $f: (0,1) \rightarrow R$ has larger Cardinality than $|R|$ [7]
- Q4. Determine if the sets are countable or uncountable [14]
 - a.) the set A of all function $g: Z_+ \rightarrow Z_+$
 - b.) The set B of all functions $f: Z_+ \rightarrow \{0,1\}$

Q5. Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: [14]

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

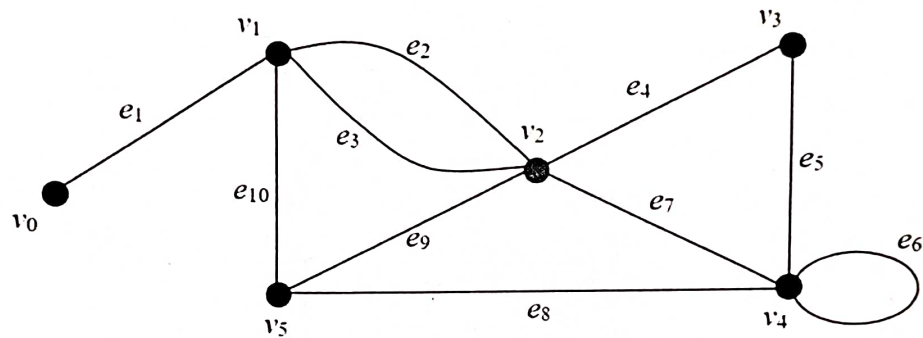
Q6. State and prove Division algorithm theorem well-ordering principle. [14]

Q7. (a) Check the validity of the following argument all integers are rational numbers. Some integers are powers of 5. Therefore, some rational numbers are powers of 5 [7]

(b) A grocery store employee is stocking apples. Each apple is a different color. There are 10 apples left in the box and the employee pulls out 2 of them at random. What is the probability that the employee pulls out one pink apple and yellow apple? [7]

Q8. Let $\Psi : G \rightarrow H$ be a homomorphism of groups. Show that if $a \in G$ has order n , then $\Psi(a) \in H$ has order dividing n . [14]

Q9. Consider the following graph



(a) Does a Hamiltonian path exist? If so describe it. If not say why not. [7]

(b) Does an Eulerian path exist? If so describe it. If not say why not. [7]