Bihar Engineering University, Patna

B.Tech. 2nd Semester Special Examination, 2024

Course: B.Tech.
Code: 105202

Subject: Mathematics-II (Probability and Statistics)

Time: 03 Hours
Full Marks: 70

Instructions:-

(i) The marks are indicated in the right-hand margin.
*(ii) There are **NINE** questions in this paper.*
*(iii) Attempt **FIVE** questions in all.*
(iv) Question No. 1 is compulsory.
(v) Assume data suitably, if not given.

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1. Choose the correct answer of the following (Any seven question only): **[2 x 7 = 14]**

(a) The mean and variance of the binomial distribution is
(i) np, npq (ii) np, np/q (iii) n/p, np/q (iv) None of these

(b) For Gamma distribution of a random variable X, which is true?
(i) $E(X) = Var(X)$ (ii) $[E(X)]^2 = Var(X)$ (iii) $E(X) = [Var(X)]^2$ (iv) None of these

(c) A fair dice is tossed 7 times. The probability that a 5 or a 6 occurs at least once is
(i) $1-\left(\frac{1}{3}\right)^7$ (ii) $1-\left(\frac{2}{3}\right)^7$ (iii) $\left(\frac{2}{3}\right)^7$ (iv) $\left(\frac{1}{3}\right)^7$

(d) If X is normal with mean 2 and standard deviation 3, then distribution of $Y = \frac{1}{2}X - 1$ is
(i) $N\left(0,\frac{9}{4}\right)$ (ii) $N\left(1,\frac{9}{4}\right)$ (iii) $N\left(0,\frac{3}{4}\right)$ (iv) $N\left(1,\frac{3}{4}\right)$

(e) If z is normally distributed with mean 0 and variance 1, then $P(z \geq -1.64)$ is
(i) 0.9495 (ii) 0.9275 (iii) 0.9385 (iv) None of these

(f) If X and Y are correlated variates each having Poisson distribution. Then $X+Y$ can be
(i) Binomial variate (ii) Poisson variate (iii) Normal variate (iv) None of these

(g) Let A and B be events with $P(A)=\frac{3}{8}, P(B)=\frac{5}{8}$ and $P(A\cup B)=\frac{3}{4}$. Then $P(A^C)$ is
(i) $\frac{5}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{1}{4}$ (iv) $\frac{2}{5}$

(h) Let E be an event for which $P(E)>0$, then for any event A
(i) $P(A/E)\geq 0$ (ii) $P(A/E)=0$ (iii) $P(A/E)=1$ (iv) None of these

(i) Let X_i be independent and identically distributed (iid) random variable such that each X_i has probability density function $f(x)=\frac{1}{\lambda}e^{-x/\lambda}$, $x>0$. Then $E(X_i)$ is
(i) λ (ii) λ^2 (iii) $n\lambda$ (iv) λ/n

(j) The Probability density function of a continuous random variate is

$$f(x)=\begin{cases} 0, & x<2 \\ \frac{3+2x}{18}, & 2\leq x\leq 4 \\ 0, & x>4 \end{cases}$$

Then, probability that x will lie between $2\leq x\leq 3$ is
(i) $\frac{4}{9}$ (ii) $\frac{2}{9}$ (iii) $\frac{5}{9}$ (iv) $-\frac{2}{9}$

Q.2 (a) In a hand at whist, what is the chance that 4 Kings are held by a specified player? **[4]**

(b) Prove that if A, B and C are random events in a sample space and if A, B, C are pairwise independent and A is independent of $(B\cup C)$, then A, B and C are mutually independent. **[10]**

Q.3 (a) In a small town of 1000 people there are 400 females and 200 colour-blind persons. It is known that ten percent i.e. 40, of the 400 females are colour-blind. Find the probability that a randomly chosen person is colour-blind, given that the selected person is a female. [7]

(b) a) A random variable (r.v.) X takes the values 1, 2, 3,........., k, with the probabilities [7]

$P[X = j] = f(j) = cj, \quad j = 1, 2, 3, ..., k$

Find the constant c such that $f(j)$ is a probability mass function.

Q.4 (a) Let X be a random variable (r.v.) with p.m.f (probability mass function) given by the following table. [7]

x:	−2	−1	0	1	2
f:	$1/10$	$2/10$	$4/10$	$2/10$	$1/10$

Compute $E(X^2)$, $E(X)$ and find $Var(X)$.

(b) Find the mean and variance of the exponential distribution [7]

$f(x) = \frac{1}{b} e^{-\frac{x}{b}}$ for $x > 0,\ b > 0.$

Q.5 (a) A random variable X has the density function $f(x) = \frac{1}{2} e^{-|x|}, -\infty < x < \infty$. Find the value of x_0 such that $F(x_0) = 0.5$. [7]

(b) Show that the function $f(x) = \frac{1}{\pi(1 + x^2)}, -\infty < x < \infty$ is a density function. [7]

Q.6 (a) Suppose X have the density function. [7]

$$f(x) = \begin{cases} xe^{-x}, & if\ x > 0 \\ 0, & otherwise \end{cases}$$

Find its moment generating function whenever it exists.

(b) If X be a binomially distributed random variable with E(X) = 2 and Var(X) = 4/3. Find the distribution of X. [7]

Q.7 (a) Suppose (X, Y) has the joint probability density function. [7]

$$f_{X,Y}(x, y) = \begin{cases} 8xy, & if\ 0 < x < y < 1 \\ 0, & otherwise \end{cases}$$

With $E(X) = \frac{8}{15}$, $E(Y) = \frac{4}{5}$. Evaluate the Covariance of X and Y.

(b) Calculate the co-efficient of correlation and obtain the lines of regression for the following data. [7]

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Q.8 (a) By the method of least squares, fit a parabolic equation $y = a + bx + cx^2$ to the following data. [10]

x	−3	−2	0	3	4
y	18	10	2	2	5

(b) Two ladies Deepti and Nancy, were asked to rank lipsticks from 7 known companies. The ranks given by them are as follows. [4]

Lipsticks companies	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Rank X by Deepti	2	1	4	3	5	7	6
Rank Y by Nancy	1	3	2	4	5	6	7

Find the Spearman's Rank correlation coefficient.

Q.9 (a) A die is thrown 276 times and results of the throws are tabulated below. [7]

Number on the die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test, if the die is unbiased using χ^2 -Test. (Given $\chi^2_{0.05} = 11.07$ at d.f.=5)

(b) The theory predicts the proportion of beans in the four groups G_1, G_2, G_3, G_4 should be in the ratio 9: 3: 3: 1. In an experiment with 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? (Given $\chi^2_{0.05} = 7.815$ at d.f.=3) [7]

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