Bihar Engineering University, Patna

B.Tech. 2nd Semester Special Examination, 2024 Time: 03 Hours Course: B. Tech. Full Marks: 70 Code: 103202 Subject: Mathematics-II (Linear Algebra, Transform Calculus and Numerical Methods) Instructions:-The marks are indicated in the right-hand margin. (ii) There are NINE questions in this paper. (iii) Attempt FIVE questions in all. (iv) Question No. 1 is compulsory. (v) Assume data suitably, if not given. Choose the correct answer of the following (Any seven question only): (a) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ satisfies the matrix equation $A^2 - kA + 2I = 0$, then what is the value (iv) 2 (iii) 1 (i) 0(ii) 3 Which method is used to approximate the solution to ordinary differential equations? (b) (ii) Newton-Raphson method (i) Simpson's rule (iv) None of these (iii) Runge-Kutta method The columns of an orthogonal matrix forms (ii) An orthonormal set of vectors (i) An orthogonal set of vectors (iii) A linearly independent set (iv) All of above (d) Define Fourier transformation (i) $\int_{-\infty}^{\infty} e^{-ist} f(t) dt$ (ii) $\int_{-\infty}^{0} e^{-ist} f(t) dt$ (iii) $\int_0^\infty e^{-st} f(t) dt$ (iv) None of these (e) What is the Poisson equation in two dimensions? (i) $\nabla^2 \varphi = \varphi(x, y)$ (ii) $\nabla^2 \varphi = 0$ (iii) $\nabla^2 \varphi = f(x, y)$ (iv) None of these (f) What is the primary objective of numerical methods in solving mathematical problems? (i) Efficiency (ii) Stability (iii) Exact solutions (iv) Accuracy If M is a 7×5 matrix of rank 3 and N is a 5×7 matrix of rank 5, then rank (MN) is (g) (ii) 1 (iii) 2 Which property of Laplace transforms allows us to easily solve linear differential (h) equations? (i) Differentiation in time (ii) Linearity (iii) Convolution (iv) None of these (i) Which numerical method is used to find the roots of nonlinear equations? (i) Bisection method (ii) Euler method (iii) Gaussian elimination (iv) None of these (j) What boundary conditions are typically used in the finite difference solution of the twodimensional Laplace equation? (i) Mixed boundary conditions (ii) Neumann boundary conditions (iii) Dirichlet boundary conditions (iv) None of these

Q.2 Find the matrix which is similar to diagonal matrix of
$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$
.

Q.3 (a) Solve
$$y'' + 4y' + 3y = e^{-t} \sin t$$
, if $y = 1$, $Dy = 1$ when $t = 0$. [7] (b) Use convolution theorem to find the inverse Laplace transform of the $\frac{16}{(S-2)(S+2)^2}$. [7]

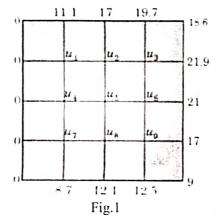
- Q.4 (a) Find the Laplace transformation of $\sin \sqrt{t}$.
 - (a) Find the Laplace transformation of $\frac{1}{s(s+1)^3}$. [5]
 - (c) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. [5]
- Q.5 (a) Prove that $A^3 4A^2 3A + 11I = 0$, where $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$. [7]
 - (b) Investigate for consistency of the following equations and if possible, find the solutions:

$$4x - 2y + 6z = 8$$

$$x + y - 3z = -1$$

$$15x - 3y + 9z = 21$$

- **Q.6** (a) If f(1.15) = 1.0723, f(1.20) = 1.0954, f(1.25) = 1.1180 and f(1.30) = 1.1401, [7] find f(1.28).
 - (b) Compute the value of $\int_{1}^{2.2} (\sin x \log x + e^x) dx$ using Simpson's 1/3rd rule. [7]
- Q.7 (a) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition y = 1 at x = 0; find y for x = 0.1 by Euler's method.
 - (b) Find an approximate value of y when x = 0.1, if $dy/dx = x y^2$ and y = 1 at [7] x = 0, using Taylor's series.
- **Q.8** Solve the Laplace equation $u_{xx} + u_{yy} = 0$ from the given Fig.1 (upto 4 iteration). [14]



- **Q.9** (a) Find a real root of the equation $x \log_{10} x = 1.2$ by Regula-Falsi method, correct 4 decimal places.

[8]

Also, explain why y_3 is not 27.