Bihar Engineering University, Patna

B.Tech. 2nd Semester Special Examination, 2024

Course: B.Tech.
Code: 102202

Time: 03 Hours
Full Marks: 70

Subject: Mathematics-II (ODE & Complex Variables)

Instructions:-

(i) The marks are indicated in the right-hand margin.
*(ii) There are **NINE** questions in this paper.*
*(iii) Attempt **FIVE** questions in all.*
(iv) Question No. 1 is compulsory.
(v) Assume data suitably, if not given.

Q. 1. Answer any seven question of the following: **[2 x 7 = 14]**

(a) Use change of order of integration to evaluate $\int_0^1\int_0^x e^{\frac{x}{y}}dxdy$.

(b) If $A = 2x^2I - 3yzJ + xz^2K$ then find $\nabla \bullet A$.

(c) Find the solution of the differential equation $(x^2 + y^2)dy = xydx$.

(d) Write the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be Exact.

(e) Write the Rodrigue's formula for $P_n(x)$.

(f) Obtain the value of the Wroskian matrix for the differential equation $\frac{d^2y}{dx^2} - y = x$.

(g) For what value of a, the function $2x - 6x^2 + ay^2$ is Harmonic ?

(h) Find the poles of $(z) = \frac{1}{sinz - cosz}$.

(i) Write the cross ratio of four points z_1, z_2, z_3, z_4.

(j) Define Taylor's series.

Q.2 (a) Find the area of the loop of the curve $3ay^2 = x(x-a)^2$. [7]

(b) Evaluate $\int_0^1\int_0^x\int_0^{x+y}(x + y + z)\, dzdydx$. [7]

Q.3 (a) Verify Green's theorem for $\int_C (x^2ydx + x^2dy)$ where C is the boundary described counter clockwise of triangle with vertices (0,0), (1,0), (0,1). [7]

(b) Find the total work done in moving a particle in the force field given by $F = 3xyI - 5zJ + 10xK$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from t=1 to t=2. [7]

Q.4 (a) Solve $xlogx\frac{dy}{dx} + y = logx^2$. [7]

(b) Solve $p(p + y) = x(x + y)$. [7]

Q.5 (a) Use method of variation of parameter to obtain the solution of $\frac{d^2y}{dx^2} + y = tanx$. [7]

(b) Solve the Cauchy's linear equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$. [7]

Q.6 (a) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. [7]

(b) Prove that $(n + 1)P_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x)$. [7]

Q.7 (a) Given $v(x, y) = x^4 - 6x^2y^2 + y^4$ find $f(z) = u(x, y) + iv(x, y)$ such that $f(z)$ is analytic. [7]

(b) Find the image of the mapping of the region $1 \leq x \leq 2$ and $2 \leq y \leq 3$ under the mapping $w = e^z$. [7]

Q.8 (a) Obtain the Laurent series which represent the function $f(z) = \frac{1}{(1+z^2)(z+2)}$ where [7]

(i) $1 < |z| < 2$ (ii) $|z| > 2$.

(b) Evaluate $\int \frac{e^z}{z^2+1} dz$ over the circular path $|z|=2$. [7]

Q.9 Write short on ***any two*** of the followings:- [7x2=14]

(a) Froenious method

(b) Singularities of an analytic function

(c) Stoke's Theorem and Gauss Divergence theorem