Bihar Engineering University, Patna
B.Tech 1st Semester Exam-2022

Course: B.Tech. *Time: 03 Hours*
Code: 105102 *Subject: Mathematics-1(Calculus & Linear Algebra)* *Full Marks: 70*

Instructions:-
(i) The marks are indicated in the right-hand margin.
(ii) There are NINE questions in this paper.
(iii) Attempt FIVE questions in all.
(iv) Question No. 1 is compulsory.

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Q.1 ***Choose the correct answer of the following (any seven Question only):*** **[2 x 7 = 14]**

(a) Lagrange's mean value theorem can be proved for a function $f(x)$ by applying Roll's mean value theorem to the function
(i) $\varphi(x) = f(x) + kx^2$ (ii) $\varphi(x) = f(x) - kx^2$
(iii) $\varphi(x) = f(x) + kx$ (iv) $\varphi(x) = \{f(x)\}^2 + kx^2$

(b) The function $f(x) = x(x+3)e^{-\frac{x}{2}}$ satisfy all conditions of Roll's mean value theorem in the interval [-3, 0]. Then the value of c is :
(i) 0. (ii) 1.
(iii) 2. (iv) - 2.

(c) If $\begin{bmatrix} 4/3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. What is the associated eigenvalue?
(i) 4/3 (ii) 5. (iii) -2. (iv) None of the above

(d) If f is continuous on [-2.354, 2.354] then
(i) $\int_{-2.354}^{2.354} f(cosx)dx = 0$ (ii) $\int_{-2.354}^{2.354} f(cosx)dx = 2\int_{0}^{2.354} f(cosx)dx$
(iii) 2.354 (iv) None of the above

(e) Let $f(x) = |x|^{\frac{3}{2}}$, $x \in R$ then
(i) f is uniformly continuous
(ii) f is continuous, but not differentiable at x = 0
(iii) f is differentiable and derivative of f is continuous
(iv) f is differentiable, but derivative of x is discontinuous at x=0

(f) If A(2) = 2i – j+2k, A(3) = 4i – 2j +3k, then $\int_2^3 A.\frac{dA}{dt}dt$ is
(i) 5 (ii) 10 (iii) 15 (iv) 20

(g) If $\nabla \times \vec{F} = 0$. Then it is called.
(i) Solenoidal (ii) Rotational (iii) Irrotational (iv) None

(h) If $3x + 2y + z = 0$, $x + 4y + z = 0$, $2x + y + 4z = 0$ be a system of equations, then
(i) It is inconsistent
(ii) It has only the trivial solution x =0, y=0, z=0
(iii) Determinant of the matrix of coefficients is zero.
(iv) None of these

(i) If f(x) is continuous in the closed interval [a,b] and f'(x) exists in (a,b) and f(a) = f(b), then there exist at least one value c(a<c<b) Such that f'(c) = 0 is called
(i) Taylor's theorem (ii) Mac Laurin's theorem
(iii) Rolle's theorem (iv) Lagrange's mean value theorem

(j) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is
(i) 3 (ii) 2
(iii) 1 (iv) None of these

P.T.O.

Q.2 (a) Show that: [7]

$$\int_0^\infty x^2 e^{-x^4} dx \times \int_\infty^\infty e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$$

(b) Use L' Hospital rule to find the following limits. [7]

$$\lim_{x\to 0} \frac{\cos x - \ln(1+x) - 1 + xy^x}{\sin^2 x}.$$

Q.3 (a) Test the convergence of [7]

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \frac{3.6.9.12}{7.10.13.16}x^4 + \cdots$$

(b) Examine the convergence of the series of which the general term is [7]

$$2^2 4^2 6^2 \ldots \frac{(2n-2)^2}{3.4.5 \ldots (2n-1)2n} x^{2n}$$

Q.4 (a) Obtain the fourth-degree Taylor's polynomial approximation to [7]
$f(x) = e^{2x}$ *about* $x = 0$. Find the maximum error when $0 \le x \le 0.5$

(b) It is given the Rolle's theorem holds the function $f(x) = x^3 + bx^2 + cx, 1 \le x \le 2$ at the Point $x = 4/3$. Find the value of b and c. [7]

Q.5 (a) Evaluate $\int_0^\infty \log\left(x + \frac{1}{x}\right) \frac{dx}{1+x^2}$. [7]

(b) Discuss the convergence of the sequence whose n-th term is $a_n = \frac{(-1)^n}{n} + 1$. [7]

Q.6 (a) If f(x) = log (1+x), x>0 using <Madaurin's theorem, show that for $0<\theta<1$, [7]

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3(1+\theta x)^3}$$

Deduce that $\log(1+x) = < x - \frac{x^2}{2} + \frac{x^3}{3}$ for x>0

(b) Using Taylor's theorem, express the polynomial $2x^3+7x^2+x-6$ in powers of (x-1). [7]

Q.7 (a) Find the values of a,b,c if $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal? [7]

(b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse. [7]
Also express $A^5-4A^4-7A^3+11A^2-A-10I$ as a linear polynomial in A.

Q.8 (a) Expand long x in power of $(x-1)$ by Taylor's theorem. [7]

(b) By using Beta function evaluate $\int_0^1 x^5(1-x^3)^{10} dx$. [7]

Q.9 (a) Find the Fourier series to represent the function defined as [7]

$$f(x) = \begin{cases} x + \frac{\pi}{2}, & -\pi < x < 0 \\ \frac{\pi}{2} - x, & 0 < x < \pi \end{cases}$$

(b) Evaluate: i) *div* $\boldsymbol{F}$ and *Curl* $\boldsymbol{F}$, where $\boldsymbol{F} = grad\,(x^3+y^3+z^3-3xyz)$. [7]

ii) If $F = (x+y+z)\,\hat{i} + \hat{j} - (x+y)\hat{k}$ *show that* $F\ Curl\ F = 0$.