QUESTION BANK

Subject: Differential Equation

First Order Ordinary Differential Equation

Question 1 *Solve*

$$3e^x \tan(y)dx + (1-e^x)\sec^2(y)dy = 0.$$

Question 2 *Solve*

$$\frac{dy}{dx} = \frac{y}{x} + \tan(\frac{y}{x}).$$

Question 3 *Solve*

$$ysin(2x)dx + (1+y^2+cos^2(x))dy = 0.$$

Question 4 *Solve*

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0.$$

Question 5 *Solve*

$$(x+2y^3)(\frac{dy}{dx}) = y.$$

Question 6 *solve*

$$(1+x+xy^2)dy - (y+y^3)dx = 0.$$

Question 7 *Solve*

$$x\frac{dy}{dx} + ylog(y) = xye^x.$$

Question 8 *Solve*

$$\frac{dy}{dx} + \frac{1}{x}sin(2y) = x^2cos^2y.$$

First order but not first degree ordinary differential equation

Question 9 *Solve*

$$4xp^2 = (3x-a)^2.$$

Question 10 *Solve*

$$p(p+x) = y(y+x).$$

Question 11 *Solve*

$$x = py + ap^2.$$

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Question 12 *Solve*

$$y = 2px + p^2y.$$

Question 13 *Solve*

$$sin(px)cos(y) = cos(px)sin(y) + p.$$

Question 14 *Solve*

$$p = log(px - y).$$

Second order ordinary differential equation

Question 15 *Solve*

$$y'' - 2y' + y = x^2e^{3x}.$$

Question 16 *Solve*

$$x^2y'' + y = 3x^2.$$

Question 17 *Solve by method of variation of parameters*

$$y'' + y = sec(nx).$$

Question 18 *Find the series solution of*

$$4xy'' + 2y' + y = 0.$$

Question 19 *Solve the Legendre Differential equation*

$$(1 - x^2)y'' - 2xy + n(n+1)y = 0.$$

Question 20 *Prove that*

$$(n+1)P_{n+1} = (2n+1)xP_n - nP_{n-1}.$$

Question 21 *Prove that*
(i)

$$J_{-1/2}(x) = \sqrt{(2/\pi x)}cos(x).$$

(ii)

$$J_{1/2}(x) = \sqrt{(2/\pi x)}sin(x).$$

Question 22 *Prove that*

$$\frac{d}{dx}\{x^nJ_n(x)\} = x^nJ_{n-1}(x).$$

****** The End ******

Partial differential equation

Question 23 *Form the partial differential equation by eliminating the function f and g from*
(i)

$$z = f(x + iy) + g(x - iy).$$

(ii)

$$z = yf(x) + xg(y).$$

Question 24 *Form the partial differential equation by eliminating the function f from*

$$f(x^2 + y^2 + z^2, z^2 - 2xy) = 0.$$

Question 25 *Solve the partial differential equation*

$$ptan(x) + qtan(y) = tan(z).$$

Question 26 *Solve the partial differential equation*

$$p - 2q = 3x^2 sin(y + 2x).$$

Question 27 *Solve the partial differential equation*

$$(y - zx)p + (x + yz)q = x^2 + y^2.$$

Question 28 *Solve the partial differential equation*

$$(y + z)p + (z + x)q = x + y.$$

Question 29 *Solve the partial differential equation*

$$x_2x_3p_1 + x_3x_1p_2 + x_1x_2p_3 + x_1x_2x_3 = 0.$$

Non-Linear Partial differential equation

Question 30 *Solve the non-linear partial differential equation*

$$p = (z + qy)^2.$$

Question 31 *Solve the non-linear partial differential equation*

$$(p^2 + q^2)y = qz.$$

Question 32 *Solve the non-linear partial differential equation*

$$pq = px + qy.$$

Question 33 *Solve the non-linear partial differential equation*

$$pq = px + qy.$$

Question 34 *Solve the non-linear partial differential equation*

$$z = px + qy + pq.$$

Homogeneous and non-homogeneous Partial differential equation

Question 35 *Solve*

$$(D^2 + 3DD' + 2D'^2)z = x + y.$$

Question 36 *Solve*

$$(D^3 - 6D^2D' + 11DD'^2 - 6D'^3)z = e^{5x+6y}.$$

Question 37 *Solve*

$$(D^2 - 5DD' + 4D'^2)z = sin(4x + y).$$

Question 38 *Solve*

$$DD'(D - 2D' - 3)z = 0.$$

Question 39 *Solve*

$$(D - D' - 1)(D - D' - 2)z = e^{2x-y}.$$

Question 40 *Solve*

$$(D - D'^2)z = cos(x - 3y).$$

****** The End ******

Subject: Complex Variables

Question 41 *Show that* $\lim_{z\to 0}\frac{\bar{z}}{z}$ *does not exist.*

Question 42 *Show that* $\lim_{z\to 0}\frac{Re(z)^2}{|z|}$ *is zero.*

Question 43 *Find the limit of the following* $\lim_{z\to 1+i}\frac{z^2-z+1-i}{z^2-2z+2}$.

Question 44 *Show that the the function* $f(z)=|z|^2$ *is differentiable only at* $z=0$.

Question 45 *Examine the continuity of the function* $f(z)=\begin{cases}\frac{z^3-iz+z-i}{z-i}, & if \quad z\neq 0\\ 0, & if \quad z=0\end{cases}$ *at* $z=i$

Question 46 *If* $f(z)=\begin{cases}\frac{x^3y(y-ix)}{x^6+y^6}, & if \quad z\neq 0\\ 0, & if \quad z=0\end{cases}$ *then discuss* $\frac{df}{dz}$ *at* $z=0$

Question 47 *Prove that the function* $f(z)=|z|^2$ *is continuous everywhere but nowhere differentiable except at the origin.*

Question 48 *State the necessary and sufficient condition to be analytic.*

Question 49 *Show that the function* $e^x(cosy+isiny)$ *is an analytic function.*

Question 50 *Show that the function* $f(z)=u+iv$, *where* $f(z)=\begin{cases}\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & if \quad z\neq 0\\ 0, & if \quad z=0\end{cases}$ *satisfy the Cauchy-Riemann equation at* $z=0$.

Question 51 *Prove that* $u=x^2-y^2-2xy-2x+3y$ *is harmonic. Find a function* v *such that* $f(z)=u+iv$ *is analytic.*

Question 52 *Evaluate* $\int_0^{1+i}(x^2-iy)dz$ *along the path* $y=x$.

Question 53 *State and prove the Caucy integral formula.*

Question 54 *Evaluate* $\int_C\frac{dz}{z^2+9}$, *where* C *is the circle* $|z+3i|=2$

Question 55 *Use Cauchy Integral formula to evaluate* $\int_C\frac{z}{z^2-3z+2}dz$, *where* C *is the circle* $|z-2|=1/2$

Question 56 *Use Cauchy Integral formula to evaluate* $\int_C\frac{e^{3iz}}{z+\pi}^3dz$, *where* C *is the circle* $|z-\pi|=3.2$

Question 57 *Find the image of the triangle with vertices at* $1+i, i, 1-i$ *in* z*-plane under the transformation* $w=3z+4-2i$

Question 58 *Find the bilinear transformation which maps the points* $z=1, i, i-1$ *into the points* $w=i, 0, -i$.

Question 59 *Define the singularity of a function. Find the singularity of* $f(z) = sin(1/z)$.

Question 60 *Determine the poles of the function* $f(z) = \frac{1}{z^4+1}$

Question 61 *Find the residue of* $\frac{1}{(z^2+1)^3}$ *at* $z = i$.

Question 62 *Using Residue theorem, evaluate* $\frac{1}{2\pi i}\int_C \frac{e^{zt}dz}{z^2+2z+2}$

Question 63 *Evaluate* $\int_C \frac{1}{sinhz}dz$, *where* C *is the circle* $|z| = 4$.

Question 64 *Evaluate the integral* $\int_0^{2\pi} \frac{d\theta}{5-4cos\theta}$.

Question 65 *Use complex variable technique to find the value of the integral* $\int_0^{2\pi} \frac{d\theta}{2+cos\theta}$

****** The End ******

Subject: Numerical Methods

Question 66 *i. In Newton's forward difference formula what is u?*
i) $u = \frac{x-x_n}{h}$ *ii)* $u = x - x_n$ *iii)* $u = \frac{(x-x_n)^2}{h}$ *iv)* $u = \frac{x-x_0}{h}$.

ii. Newton's forward and backward interpolation formula is used for
i) Equal interval ii) Unequal interval iii) Both equal and unequal iv) None of these.

iii. $(\Delta - \bigtriangledown)x^2$ *is equal to*
i) h^2 *ii)* $2h^2$ *iii)* $3h^2$ *iv)* $4h^2$.
where h is step length.

iv. Compute $\Delta^3(1-2x)(1-3x)(1-4x)$
i) 0 ii) -144 iii) 1 iv) 169 .

Question 67 *Solve* $x^3 - 9x + 1$ *by Regula Falsi method using* $x_1 = 2$ *and* $x_2 = 4$.

Question 68 *Find the root of the equation* $x \tan x = 1.28$ *that lies between* 0 *and* 1 *correct upto two places of decimals, using bisection method.*

Question 69 *Find the real root of the equation* $f(x) = x^3 - x - 1 = 0$ *by bisection method up to 5th approximation.*

Question 70 *Find the positive real root of* $x - \cos x = 0$ *by bisection method correct upto four decimal places between* 0 *and* 1.

Question 71 *Find a real root of the equation* $x^6 - x^4 - x^3 - 1 = 0$ *using the method of false position up to four decimal places.*

Question 72 *Find the real root of the equation* $\log x - \cos x = 0$ *by Newton's Raphson method.*

Question 73 *The equation* $x^2 + ax + b = 0$ *has two real roots* α *and* β. *Show that the method*
(a) $x_{k+1} = \frac{1}{x_k}(ax_k + b)$ *converges to a if* $|\alpha| < |\beta|$.
(b) $x_{k+1} = -\frac{b}{x_k + \alpha}$ *converges to a if* $|\alpha| < |\beta|$.

Question 74 *Locate the intervals which contain the positive real roots of the equation* $x^3 - 3x + 1 = 0$. *Obtain these roots correct to three decimal places, using the method of false position. (Ans: 1.531956)*

Question 75 *Find the root correct to two decimal places of the equation* $xe^x = \cos x$, *using the method of false position. (Ans: 0.51692).*

Question 76 *Derive the Newton's method for finding the* q^{th} *root of a positive number* $N, N^{1/q}$, *where* $N > 0, q > 0$. *Hence, compute* $17^{1/3}$ *correct to four decimal places, assuming the initial approximation as* $x_0 = 2$. *(Ans: 2.571282)*

Question 77 *Find the interpolating polynomial for y from the following data, using Newton's forward and backward formula:*

x	4	6	8	10
y	1	3	8	16

Question 78 *The population of a town in the census is as given in the following table data:*

year(x)	*1961*	*1971*	*1981*	*1991*	*2001*
Population (in 1000's)	*46*	*66*	*81*	*93*	*101*

Estimate the population in the year 1996 using Newton's a) forward interpolation and b) backward interpolation formula.

Question 79 *Derive Newton's forward difference interpolation formula.*

Question 80 *Use Lagrange's interpolation formula to find the value of x when $y = 20$ and when $y = 40$ using the following data:*

X	:	1	2	3	4
Y	:	1	8	27	64

Question 81 *Using Newton's forward interpolation formula, find the value of $f(x)$ at $x = 1.3$ from the following data:*

x	0	1	2	3	4
$f(x)$	1	1.5	2.2	3.1	4.3

Question 82 *Compute the value of $f(x)$ for $x = 2.5$ from the table*

x	:	1	2	3	4
$f(x)$	:	1	8	27	64

using Lagrange,s interpolation.

Question 83 *Find $f(x)$ as a polynomial in x for the following data by Newton's interpolation formula*

x	-2	-1	0	1	2	3
$f(x)$	9	16	17	18	44	81

Hence, interpolate at $x = 0.5$ and $x = 3.1$.

Question 84 *Find the Lagrange interpolating polynomial that fits the following data*

x	-2	-1	0	2	4
$f(x)$	3	-3	1	-2	6

Hence, interpolate at $x = 0.5$ and $x = 3.1$.

Question 85 *Compute $\Delta^3(1-2x)(1-3x)(1-4x)$. (Ans: -144)*

Question 86 *For the data*

x	-2	-1	0	1	2	3
$f(x)$	15	5	1	3	11	25

construct the forward difference formula. Hence, find $f(0.5)$.(Ans: 1.25)

Question 87 *For the following data, calculate the differences and obtain the Newton's forward and backward difference interpolation polynomials. Are these polynomials different? Interpolate at* $x = 0.25$ *and* $x = 0.35$.

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

Question 88 *Using Newton's backward difference interpolation, interpolate at* $x = 1.0$ *from the following data.*

x	0.1	0.3	0.5	0.7	0.9	1.1
$f(x)$	-1.699	-1.073	-0.375	-0.443	1.429	2.631

Question 89 *Following is the table of half yearly premium of the policies maturing at different ages. Estimate the premium of policies maturing at age of* 63

Age	45	50	55	60	65
Premium (in Rs.)	114.84	96.16	83.32	74.48	68.48

Question 90 *Prove the followings*

- $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$
- $(1 + \Delta)(1 - \nabla) = 1$
- $\Delta - \nabla = -\Delta\nabla$
- $\mu\delta = (\Delta + \nabla)/2$
- $\Delta\left(\frac{2^x}{(x+1)!}\right); h = 1$

Question 91 *The function* $y = f(x)$ *is given in the points* $(7, 3), (8, 1), (9, 1)$ *and* $(10, 9)$. *Find the value of* y *for* $x = 9.5$ *using Lagrange interpolation formula.*

Question 92 *A third degree polynomial passes through the points* $(0, -1), (1, 1), (2, 1)$ *and* $(3, -2)$. *Find the polynomial.*

Question 93 *Evalute* $\int_0^{\pi/2} \sqrt{\sin x} dx$.
Taking $n = 6$, *correct upto* 4 *significant figures by i) simpson's* $\frac{1}{3}$ *rule. ii) Trapezoidal rule.*

Question 94 *Evaluate* $\int_{30^0}^{90^0} \log_{10}(\sin x) dx$
By Simpson's one third rule by dividing the interval into 6 *parts.*

Question 95 *A river is* 80*m wide. the depth 'y' of the river at a distance* x *from one bank is given by the following table*

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find approximation the area of cross section of the river using Simpson's one third rule.

Question 96 *Find the soltuion of the following initial value problem by using Euler's method.* $\frac{dy}{dx} + 2y = 0, y(0) = 1$

Question 97 *Solve the boudary value problem* $y'' - 64y + 10 = 0, y(0) = y(1) = 0$ *by the finite-difference method. compute the value with value of* $y(0.5)$ *and compare it with the true value.*

****** The End ******