Mathematics-II (Numerical Methods)
Lecture Notes
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by

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The Adams-Bashforth predictor-corrector method is given by
Predictor P: Adams-Bashforth method of fourth order.

$$y_{i+1}^{(p)} = y_i + \frac{h}{24}\left[55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3}\right].$$

The method requires the starting values y_i, y_{i-1}, y_{i-2} and y_{i-3}.
Corrector C: Adams-Moulton method of fourth order.

$$y_{i+1}^{(c)} = y_i + \frac{h}{24}\left[9f(x_{i+1}, y_{i+1}^{(p)}) + 19f_i - 5f_{i-1} + f_{i-2}\right].$$

The method requires the starting values y_i, y_{i-1}, y_{i-2}.

Example

Using the Adams-Bashforth predictor-corrector equations, evaluate $y(1.4)$, if y satisfies $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and $y(1) = 1, y(1.1) = 0.996, y(1.2) = 0.986, y(1.3) = 0.972.$

Solution: Adams-Bashforth method of fourth order.

$$y_{i+1}^{(p)} = y_i + \frac{h}{24}\left[55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3}\right].$$

The method requires the starting values y_i, y_{i-1}, y_{i-2} and y_{i-3}.

Corrector C: Adams-Moulton method of fourth order.

$$y_{i+1}^{(c)} = y_i + \frac{h}{24}\left[9f(x_{i+1}, y_{i+1}^{(p)}) + 19f_i - 5f_{i-1} + f_{i-2}\right].$$

The method requires the starting values y_i, y_{i-1}, y_{i-2}.

We have $f(x,y) = \frac{1}{x^2} - \frac{y}{x}$, with $h = 0.1$, we are given the values

$y(1) = 1, y(1.1) = 0.996, y(1.2) = 0.986, y(1.3) = 0.972.$

Predictor application

For $i = 3$, we obtain

$$y_4^{(p)} = y_3 + \frac{h}{24}\left[55f_3 - 59f_2 + 37f_1 - 9f_0\right].$$

We have

$$f_0 = f(x_0, y_0) = f(1, 1) = 1 - 1 = 0, f_1 = f(x_1, y_1) = f(1.1, 0.996) = -0.079008,$$

$$f_2 = f(x_2, y_2) = f(1.2, 0.986) = -0.127222, f_3 = f(x_3, y_3) = f(1.3, 0.972) = -0.155976.$$

$$y_4^{(0)} = y_4^{(p)} = 0.972 + \frac{0.1}{24}[55(-0.155976) - 59(-0.127222) + 37(-0.079008) - 9(0)] = 0.955351.$$

Corrector application

First iteration For $i = 3$, we get

$$y_4^{(c)} = y_4^{(1)} = y_3 + \frac{h}{24}\left[9f(x_4, y_4^{(0)}) + 19f_3 - 5f_2 + f_1\right].$$

Now, $f(x_4, y_4^{(0)}) = f(1.4, 0.955351) = -0.172189.$

$$y_4^{(1)} = 0.972 +$$

$$\frac{0.1}{24}[9(-0.172189) + 19(-0.155976) - 5(-0.127222) + (-0.07900$$

$$0.955516.$$

Second iteration

$$y_4^{(2)} = y_3 + \frac{h}{24}\left[9f(x_4, y_4^{(1)}) + 19f_3 - 5f_2 + f_1\right].$$

Now, $f(x_4, y_4^{(1)}) = f(1.4, 0.955516) = -0.172307.$

$$y_4^{(1)} = 0.972 +$$

$$\frac{0.1}{24}[9(-0.172307) + 19(-0.155976) - 5(-0.127222) + (-0.07900$$

$$0.955512.$$

Now, we have
$|y_4^{(2)} - y_4^{(1)}| = |0.955512 - 0.955516| = 0.000004.$
Therefore, $y(1.4) = 0.955512$. The result is correct to five decimal places.

Thanks !!!