Mathematics-II (Numerical Methods)
Lecture Notes
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by

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Consider a first order initial value problem defined as

$$y' = f(x, y), \qquad y(x_0) = y_0$$

The Euler's method is defined as

$$y_{n+1} = y_n + hf(x_n, y_n).$$

Example

Solve the initial value problem $yy' = x, \quad y(0) = 1$, using the Euler method in $0 \leq x \leq 0.8$, with $h = 0.2$ and $h = 0.1$. Compare the results with the exact solution at $x = 0.8$..

Solution: We have $y' = f(x, y) = \dfrac{x}{y}$. The Euler's method gives

$$y_{n+1} = y_n + hf(x_n, y_n) = y_n + h\left(\frac{x_n}{y_n}\right).$$

Here $h = 0.2, x_0 = 0, y_0 = 1$. Now we have

$$y_1 = y(x_1) = y(0.2) = y_0 + h\left(\frac{x_0}{y_0}\right) = 1 + (0.2)(0) = 1.0$$

$$y_2 = y(x_2) = y(0.4) = y_1 + h\left(\frac{x_1}{y_1}\right) = 1 + (0.2)\left(\frac{0.2}{1.0}\right) = 1.04$$

$$y_3 = y(x_3) = y(0.6) = y_2 + h\left(\frac{x_2}{y_2}\right) = 1.04 + (0.2)\left(\frac{0.4}{1.04}\right) = 1.11692$$

$$y_4 = y(x_4) = y(0.8) = y_3 + h\left(\frac{x_3}{y_3}\right) = 1.11692 + (0.2)\left(\frac{0.6}{1.11692}\right) = 1.22436.$$

When $h = 0.1$, we have

$$y_1 = y(x_1) = y(0.1) = y_0 + h\left(\frac{x_0}{y_0}\right) = 1 + (0.2)(0) = 1.0$$

$$y_2 = y(x_2) = y(0.2) = y_1 + h\left(\frac{x_1}{y_1}\right) = 1 + (0.1)\left(\frac{0.1}{1.0}\right) = 1.01.$$

$$y_3 = y(x_3) = y(0.3) = y_2 + h\left(\frac{x_2}{y_2}\right) = 1.01 + (0.1)\left(\frac{0.2}{1.01}\right) = 1.02980.$$

$$y_4 = y(x_4) = y(0.4) = y_3 + h\left(\frac{x_3}{y_3}\right) =$$

$$1.02980 + (0.1)\left(\frac{0.3}{1.02980}\right) = 1.05893.$$

$$y_5 = y(x_5) = y(0.5) = y_4 + h\left(\frac{x_4}{y_4}\right) =$$

$$1.05893 + (0.1)\left(\frac{0.4}{1.05893}\right) = 1.09670.$$

$$y_6 = y(x_6) = y(0.6) = y_5 + h\left(\frac{x_5}{y_5}\right) =$$

$$1.09670 + (0.1)\left(\frac{0.5}{1.09670}\right) = 1.14229.$$

$$y_7 = y(x_7) = y(0.7) = y_6 + h\left(\frac{x_6}{y_6}\right) =$$

$$1.14229 + (0.1)\left(\frac{0.6}{1.14229}\right) = 1.19482.$$

$$y_8 = y(x_8) = y(0.8) = y_7 + h\left(\frac{x_7}{y_7}\right) =$$

$$1.19482 + (0.1)\left(\frac{0.7}{1.19482}\right) = 1.25341.$$

The exact solution is $y = \sqrt{x^2 + 1}$. At $x = 0.8$, the exact value is $y(0.8) = \sqrt{1.64} = 1.28062$.
The magnitudes of the errors in the solutions are the following:

$$h = 0.2 : |1.28062 - 1.22436| = 0.05626.$$

$$h = 0.1 : |1.280621.25341| = 0.02721.$$

Quiz

Question 1: You are given the differential equation $y' = 6x$ where $y = 2$ for $x = 0$. The statement: $y = 2$ for $x = 0$ is called .....

Question 2: Solve the initial value problem $yy' = x, \quad y(0) = 1$, using the Euler method in $0 \leq x \leq 0.8$, with $h = 0.2$ and $h = 0.1$. Compare the results with the exact solution at $x = 0.8$.

Thanks !!!