Mathematics-II (Numerical Methods)
Lecture Notes
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by

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In Simpson's $3/8$ rule, the number of subintervals is $n = 3N$. Hence, we have

$$h = \frac{b-a}{3N}.$$

and *Simpson* $3/8$ *rule* is defined as

$$I = \int_a^b f(x)dx =$$

$$\frac{3h}{8}[f(x_0) + f(x_{3N}) + 2\{f(x_3) + f(x_6) + ... + f(x_{3N-3})\} + 3\{f(x_1) + f(x_2) + ... + f(x_{2N-2}) + f(x_{2N-1})\}].$$

Example

Evaluate $I = \int_1^2 \frac{dx}{5+3x}$ with 3 and 6 subintervals using Simpson's $3/8$ rule. Compare with the exact solution.

Solution: With $N = 3$ and 6, we have the following step lengths and nodal points.

$N = 3, \qquad h = \dfrac{b-a}{N} = \dfrac{1}{3}.$ The nodes are $1, 4/3, 5/3, 2.0.$

We have the following tables of values.

x	1	$4/3$	$5/3$	2.00
$f(x)$	0.125	0.11111	0.10000	0.09091

Now, we compute the value of the integral.

$$\begin{aligned} I_1 = \int_1^2 \frac{dx}{5+3x} &= \frac{3h}{8}\left[f(1.0) + f(2.0) + 3\left\{f(4/3) + f(5/3)\right\}\right] \\ &= 0.125\left[0.125 + 0.09091 + 3\left\{0.11111 + 0.10000\right\}\right]. \\ &= 0.10616. \end{aligned}$$

$N = 6, \qquad h = \frac{b-a}{N} = \frac{1}{6}$. The nodes are

$$1, 7/6, 8/6, 9/6, 10/6, 11/6, 2.0.$$

We have the following tables of values.

x	1	7/6	8/6	9/6	10/6	11/6	2.00
$f(x)$	0.125	0.11765	0.11111	0.10526	0.10000	0.09524	0.09091

Now, we compute the value of the integral.

$$\begin{aligned} I_2 &= \int_1^2 \frac{dx}{5+3x} \\ &= \frac{3h}{8}\left[f(1.0)+f(2.0)+2\left\{f(9/6)\right\}+3\left\{f(7/6)+f(8/6)+f(10/6)+f(11/6)\right\}\right] \\ &= \frac{1}{16}\left[0.125+0.09091+2\left\{0.10526\right\}+3\left\{0.11765+0.11111+0.10000+0.09524\right\}\right]. \\ &= 0.10615. \end{aligned}$$

The exact value of the integral is

$$I = \int_1^2 \frac{dx}{5+3x} = \frac{1}{3}\left[\ln(5+3x)\right]_1^2 = \frac{1}{3}\left[\ln(11)-\ln(8)\right] = 0.10615$$

The errors in the solutions are the following:

$$|Exact - I_1| = |0.10615 - 0.10616| = 0.00001.$$

$$|Exact - I_2| = |0.10615 - 0.10615| = 0.00000.$$

The magnitude of the error for $N = 3$ is 0.00001 and for $N = 6$ the result is correct to all places.

Remarks: The Simpson $1/3$ rule and Simpson $3/8$ rule produces exact results for polynomials of degree ≤ 3.

Quiz

Question 1: Find the approximate value of $I = \int_1^2 \frac{dx}{5+3x}$, using the Simpson $1/3$ rule with 4 and 8 equal subintervals. Using the exact solution, find the absolute errors.

Question 2: What are the disadvantages of the Simpson's $3/8$ rule compared with the Simpson's $1/3$ rule?

Question 3: What is the restriction in the number of nodal points, required for using the Simpson's $3/8$ rule for integrating $I = \int_a^b f(x)dx$?

Thanks !!!