Mathematics-II (Numerical Methods)
Lecture Notes
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by

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Simpson $1/3$ rule: We subdivide the given interval [a, b] into even number of subintervals of equal length h. That is, we obtain an odd number of nodal points. We take the even number of intervals as $2N$. The step length is given by $h = (b-a)/(2N)$. The nodal points are given by $a = x_0, x_1 = x_0 + h, x_2 = x_0 + 2h, ..., x_{2N} = x_0 + 2Nh = b$. Then, Simpson $1/3$ rule is defined as

$$I = \int_a^b f(x)dx =$$
$$\frac{h}{3}[f(x_0) + f(x_{2N}) + 4\{f(x_1) + f(x_3) + ... + f(x_{2N-1})\} +$$
$$2\{f(x_2) + f(x_4) + ... + f(x_{2N-2})\}].$$

Example

Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$, using the Simpson's $1/3$ rule with 2, 4 and 8 equal subintervals. Using the exact solution, find the absolute errors.

Solution: With $n = 2N = 2$ or $N = 1$ we have the following step lengths and nodal points.

$$\text{For } N = 1, \qquad h = \frac{b-a}{2N} = \frac{1-0}{2} = 0.5, \text{ The nodes are } 0, 0.5, 1.0.$$

We have the following tables of values.

x	0	0.5	1.0
$f(x)$	1	0.666667	0.5

Now, we compute the value of the integral.

$$\begin{aligned} I_1 &= \int_0^1 \frac{dx}{1+x} \\ &= \frac{h}{3}\left[f(0)+f(1.0)+4\left\{f(0.5)\right\}\right] \\ &= \frac{0.5}{3}\left[1.0+0.5+4\left\{0.666667\right\}\right]. \\ &= 0.674444. \end{aligned}$$

Again, with $n = 2N = 4$ or $N = 2$ we have the following step lengths and nodal points.

For $N = 2$, $\quad h = \dfrac{b-a}{2N} = \dfrac{1-0}{4} = 0.25$, The nodes are $0, 0.25, 0.5, 0.75, 1.0$.

We have the following tables of values.

x	0	0.25	0.5	0.75	1.0
$f(x)$	1	0.8	0.666667	0.571429	0.5

Now, we compute the value of the integral.

$$\begin{aligned} I_2 &= \int_0^1 \frac{dx}{1+x} \\ &= \frac{h}{3}\left[f(0)+f(1.0)+4\left\{f(0.25)+f(0.75)\right\}+2\left\{f(0.5)\right\}\right] \\ &= \frac{0.5}{3}\left[1.0+0.5+4\left\{0.8+0.571429\right\}+2(0.666667)\right]. \\ &= 0.693254. \end{aligned}$$

Finally, with $n = 2N = 8$ or $N = 4$ we have the following step lengths and nodal points. For

$$N = 4, \qquad h = \frac{b-a}{2N} = \frac{1-0}{8} = 0.125,$$

The nodes are $0, 0.125, 0.250, 0.375, 0.5, 0.625, 0.75, 0.875, 1.0$.

We have the following tables of values.

x	0	0.125	0.250	0.375	0.500	0.675
$f(x)$	1	0.888889	0.8	0.727273	0.666667	0.615385

Now, we compute the value of the integral.

$$\begin{aligned} I_3 &= \int_0^1 \frac{dx}{1+x} \\ &= \frac{h}{3}\left[f(0)+f(1.0)+4\left\{f(0.125)+f(0.375)+f(0.675)+f(0.875)\right\}\right. \\ &+ \left. 2\left\{f(0.25)+f(0.5)+f(0.75)\right\}\right. \end{aligned}$$

(1)

$$= \frac{0.5}{3}\left[1.0+0.5+4\left\{0.888889+0.727273+0.615385+0.533333\right\}\right.$$

$$\left.+2\left\{0.8+0.666667+0.571429\right\}\right.$$

$=0.693155.$

The exact value of the integral is $I = \ln 2 = 0.693147$. The errors in the solutions are the following:

$$|Exact - I_1| = |0.693147 - 0.694444| = 0.001297.$$

$$|Exact - I_2| = |0.693147 - 0.693254| = 0.000107.$$

$$|Exact - I_3| = |0.693147 - 0.693155| = 0.000008.$$

Example

The velocity of a particle which starts from rest is given by the following table.

$t(sec)$	0	2	4	6	8	10	12	14	16	18	20
$v(ft/sec)$	0	16	29	40	46	51	32	18	8	3	0

Evaluate using Simpson's $1/3$ rule, the total distance traveled in 20 seconds.

Solution: From the definition, we have

$$v = \frac{ds}{dt} \text{ or } s = \int v dt$$

Starting from rest, the distance traveled in 20 seconds is

$$s = \int_0^{20} v dt$$

The step length is $h = 2$.

Using the Simpson's rule, we obtain

$$\begin{aligned} s &= \int_0^{20} v\,dt \\ &= \frac{h}{3}\left[f(0)+f(20)+4\left\{f(2)+f(6)+f(10)+f(14)+f(18)\right\}\right. \\ &+ \left. 2\left\{f(4)+f(8)+f(12)+f(16)\right\}\right. \end{aligned}$$

(0)

$$= \frac{2}{3}\left[0+0+4\left\{16+40+51+18+3\right\}\right.$$

$$+2\left.\left\{29+46+32+8\right\}\right.$$

=494.667 feet .

Thanks !!!