Mathematics-II (Numerical Methods)
Lecture Notes
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by

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Definition

Newton Divided Difference Interpolation Let the $(x_i, f(x_i)), i = 0, 1, 2, , n$ be given unequal spaced data. We define the Newton divided difference interpolation formula as follows:

$$f(x) = f(x_0)+(x-x_0)f[x_0, x_1]+(x-x_0)(x-x_1)f[x_0, x_1, x_2]+ \\ ... + (x - x_0)(x - x_1)...(x - x_{n-1})f[x_0, x_1, x_2, ..., x_n].$$

Example

Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference formula

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

Solution: We form the divided difference table for the given data.

x	$f(x)$	$first \quad D.D.$	$Second \quad D.D.$	$Third \quad D.D.$	$Fourth$
-4	1245	$\frac{33-1245}{-1-(-4)} = -404$			
-1	33	$\frac{5-33}{0-(-1)} = -28$	$\frac{-28-(-404)}{0-(-4)} = 94$	$\frac{10-(94)}{2-(-4)} = -14$	
0	5	$\frac{9-5}{2-0} = 2$	$\frac{2-(-28)}{2-(-1)} = 10$	$\frac{88-10}{5-(-1)} = 13$	$\frac{13-(-1}{5-(-4}$
2	9	$\frac{1335-9}{5-2} = 442$	$\frac{442-(2)}{5-0} = 88$		
5	1335				

Table: Divided Difference Table

The Newton's divided difference formula gives

$$
\begin{aligned}
f(x) &= f(x_0) + (x - x_0) f[x_0, x_1] \\
&+ (x - x_0)(x - x_1) f[x_0, x_1, x_2] \\
&+ (x - x_0)(x - x_1)(x - x_2) f[x_0, x_1, x_2, x_3] \\
&+ (x - x_0)(x - x_1)(x - x_2)(x - x_3) f[x_0, x_1, x_2, x_3, x_4]. \\
&= 1245 + (x + 4)(-404) + (x + 4)(x + 1)(94) \\
&+ (x + 4)(x + 1)x(-14) \\
&+ (x + 4)(x + 1)x(x - 2)(3). \\
&= 1245 - 404x - 1616 + (x^2 + 5x + 4)(94) \\
&+ (x^3 + 5x^2 + 4x)(-14) \\
&+ (x^4 + 3x^3 - 6x^2 - 8x)(3). \\
&= 3x^4 - 5x^3 + 6x^2 - 14x + 5.
\end{aligned}
$$

Example

Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference formula

x	1	3	4	5	7	10
$f(x)$	3	31	69	131	351	1011

Hence, interpolate at $x = 3.5$ and $x = 8.0$. Also find, $f'(3)$ and $f''(1.5)$.

Solution: We form the divided difference table for the given data.

x	$f(x)$	$first\ \ D.D.$	$Second\ \ D.D.$	$Third\ \ D.D.$	$Fourth\ \ D.D.$	$Fifth$
1	3					
		14				
3	31		8			
		38		1		
4	69		12		0	
		62		1		0
5	131		16		0	
		110		1		
7	351		22			
		220				
10	1011					

Table: Divided Difference Table

The Newton's divided difference formula gives

$$
\begin{aligned}
f(x) &= f(x_0) + (x - x_0) f[x_0, x_1] \\
&+ (x - x_0)(x - x_1) f[x_0, x_1, x_2] \\
&+ (x - x_0)(x - x_1)(x - x_2) f[x_0, x_1, x_2, x_3] \\
&+ (x - x_0)(x - x_1)(x - x_2)(x - x_3) f[x_0, x_1, x_2, x_3, x_4] \\
&+ (x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4) f[x_0, x_1, x_2, x_3, x_4, x_5]. \\
&= 3 + (x - 1)(14) + (x - 1)(x - 3)(8) + (x - 1)(x - 3)(x - 4)(1). \\
&= 3 + 14x - 14 + 8x^2 - 32x + 24 + x^3 - 8x^2 + 19x - 12 \\
&= x^3 + x + 1.
\end{aligned}
$$

Hence $f(3.5) = P_3(3.5) = (3.5)3 + 3.5 + 1 = 47.375$, and
$f(8.0) = P_3(8.0) = (8.0)3 + 8.0 + 1 = 521.0$.
Now, $P_3'(x) = 3x^2 + 1$, and $P_3''(x) = 6x$.
Therefore,
$f'(3) = P_(3) = 3(9)+1 = 28, f''(1.5) = P_3''(1.5) = 6(1.5) = 9.$

Thanks !!!