

Lok Nayak Jai Prakash Institute of Technology Chapra, Bihar-841302

Relation between finite difference operator...

> Dr. G.K. Prajapati

LNJPIT, Chapra

Relation Between finit difference operator Mathematics-II (Numerical Methods) Lecture Notes May 13, 2020

by

Dr. G.K.Prajapati Department of Applied Science and Humanities LNJPIT, Chapra, Bihar-841302

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Relation Between finite difference operator

Definition

Central difference operator δ : The central difference operator is defined as

$$\delta f(x) = f(x + \frac{h}{2}) - f(x - \frac{h}{2})$$

Definition

Average (Mean) operator μ : The central difference operator is defined as

$$uf(x) = \frac{f(x+\frac{h}{2}) + f(x-\frac{h}{2})}{2}$$

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Prove that: (i)
$$\Delta = E - 1$$
 (ii) $\Delta - \nabla = \nabla \Delta$
(iii) $(1 + \Delta)(1 - \nabla) = 1$
(iv) $\mu = \frac{1}{2} \left[E^{1/2} + E^{-1/2} \right]$ (v) $\delta = \nabla (1 - \nabla)^{-1/2}$
(vi) $\mu = \left[1 + \frac{\delta^2}{4} \right]^{1/2}$
(vii) $E = e^{hD}$, where $D = \frac{d}{dx}$.

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Relation Between finite difference operator **Proof:** (i) We know that Ef(x) = f(x+h). Therefore,

$$\Delta f(x) = f(x+h) - f(x) \tag{1}$$

$$\implies \Delta f(x) = Ef(x) - f(x) \tag{2}$$

$$\implies \Delta = E - 1$$
 (3)

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(*ii*) L.H.S

$$(\Delta - \nabla)f(x) = \Delta f(x) - \nabla f(x)$$

$$= \{f(x+h) - f(x)\} - \{f(x) - f(x-h)\}$$

$$= f(x+h) - 2f(x) + f(x-h)$$

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R.H.S

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$$\begin{aligned} (\nabla \Delta)f(x) &= \nabla \{\Delta f(x)\} \\ &= \nabla \{f(x+h) - f(x)\} \\ &= \{\nabla f(x+h) - \nabla f(x)\} \\ &= \{(f(x+h) - f(x)) - (f(x) - f(x-h)\} \\ &= f(x+h) - 2f(x) + f(x-h) \end{aligned}$$

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Thus L.H.S. = R.H.S.



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(*iii*) We know that
$$\nabla f(x) = f(x) - f(x - h) \implies \nabla f(x) = f(x) - E^{-1}f(x) \implies \nabla = 1 - E^{-1} \implies 1 - \nabla = E^{-1}$$
 and $\Delta = E - 1 \implies 1 + \Delta = E$. Therefore,

$$(1 + \Delta)(1 - \nabla) = (E)(E^{-1}) = 1$$

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(iv) We know that

$$\mu f(x) = \left[\frac{1}{2} f(x + \frac{h}{2}) + f(x - \frac{h}{2}) \right]$$
$$= \frac{1}{2} \left[E^{1/2} f(x) + E^{-1/2} f(x) \right]$$
$$\mu f(x) = \frac{1}{2} \left[E^{1/2} + E^{-1/2} \right] f(x).$$

(4)

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$$(v)$$
 We know that $1-\nabla=E^{-1}{\rm Therefore},$

$$R.H.S. = \nabla (1 - \nabla)^{-1/2} f(x)$$

= $\nabla (E^{-1})^{-1/2} f(x)$
= $\nabla \left\{ (E^{1/2}) f(x) \right\} = \nabla f(x + \frac{h}{2})$
= $f(x + \frac{h}{2}) - f(x + \frac{h}{2} - h)$
= $f(x + \frac{h}{2}) - f(x + \frac{-h}{2})$
= $\delta f(x) = L.H.S.$

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(vi) Try Yourself.(v) We know that the Taylor's series

Ef(x)

$$= f(x+h)$$

$$= f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \dots$$

$$= f(x) + hDf(x) + \frac{h^2}{2}D^2f(x) + \dots$$

$$= e^{\left[1+hD+\frac{h^2}{2}D^2+\dots\right]}f(x)$$

$$Ef(x) = e^{hD}f(x).$$

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Thanks !!!

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