

Lok Nayak Jai Prakash Institute of Technology Chapra, Bihar-841302

finite difference operator...

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Difference Operator

Interpolation with equally spaced data Mathematics-II (Numerical Methods) Lecture Notes May 13, 2020

by

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Interpolation with equally spaced data Let the data $(x_i, f(x_i))$ be given with uniform spacing, that is, the nodal points are given by $x_i = x_0 + ih, i = 0, 1, 2, ..., n$. Now we define several finite difference operators and relation between these finite difference operators. **Notation:** We use the following notations as follows:

$$\begin{array}{l} x_0, x_1 = x_0 + h, x_2 = x_0 + 2h, ..., x_i = x_0 + ih, \text{ and} \\ f_0 = f(x_0), f_1 = f(x_1), f_2 = f(x_2), ..., f_i = f(x_i), ... \end{array}$$

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Definition

Shift Operator *E***:** The Shift operator *E* is defined as

$$Ef(x) = f(x+h)$$

In particular,

$$Ef(x_0) = f(x_0 + h) = f(x_1), Ef(x_1) = f(x_0 + 2h) = f(x_2), \dots, Ef(x_i) = f(x_0 + (i+1)h) = f(x_{i+1}), \dots$$

Therefore, the operator E when applied on f(x) shifts it to the value at the next nodal point. We have

$$E^{2}f(x) = E(Ef(x)) = E(f(x+h)) = f(x+2h).$$

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In general, we have

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Interpolation with equally spaced data $E^k f(x) = f(x + kh)$, where k is any real number.

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For example: $E^{1/2}[f(x)] = f(x + \frac{1}{2}h).$



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Definition

Forward Operator Δ : The forward operator Δ is defined as

$$\Delta f(x) = f(x+h) - f(x)$$

In particular,

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$$\Delta f(x_0) = f(x_0 + h) - f(x_0) = f(x_1) - f(x_0),$$

$$\Delta f(x_1) = f(x_0 + 2h) - f(x_0 + h) = f(x_2) - f(x_1),$$

 $\Delta f(x_i) = f(x_0 + (i+1)h) - f(x_0 + ih) = f(x_{i+1}) - f(x_i),$

These differences are called the first forward differences.



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Interpolation with equally spaced data The second forward difference is defined by

$$\begin{aligned} \Delta^2 f(x) &= \Delta \left(\Delta f(x) \right) = \Delta \left(f(x+h) - f(x) \right) = \Delta f(x+h) - A \\ &= \left\{ f(x+2h) - f(x+h) \right\} - \left\{ f(x+h) - f(x) \right\} \\ &= f(x+2h) - 2f(x+h) + f(x) \end{aligned}$$

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Interpolation with equally spaced data The forward differences can be written in a tabular form as in Table 1

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_0	$f(x_0)$			
		$\Delta f(x_0) =$		
		$f(x_1) - f(x_0)$		
			$\Delta^2 f(x_0) =$	
x_1	$f(x_1)$		$\Delta f(x_1) - \Delta f(x_0)$	
		$\Delta f(x_1) =$		$\Delta^3 f(x_0)$
		$f(x_2) - f(x_1)$		$\Delta^2 f(x_1) - \Delta$
			$\Delta^2 f(x_1) =$	
x_2	$f(x_2)$		$\Delta f(x_2) - \Delta f(x_1)$	<
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Example

Construct the forward difference table for the data

x	-1	0	1	2
f(x)	-8	3	1	12

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Interpolation with equally spaced data Solution: We have the following difference table:

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
-1	-8			
		3 - (-8) = 11		
0	3		-2 - 11 = -13	
		1 - 3 = -2		13 + 13 = 26
1	1		11 + 2 = 13	
		12 - 1 = 11		
2	12			

Table: Forward Difference Table

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Definition

Backward Difference Operator $\nabla {\bf :}$ The Backward difference operator ∇ is defined as

$$\nabla f(x) = f(x) - f(x - h)$$

In particular,

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$$\nabla f(x_1) = f(x_0 + h) - f(x_0) = f(x_1) - f(x_0),$$

$$\nabla f(x_2) = f(x_0 + 2h) - f(x_0 + h) = f(x_2) - f(x_1),$$

$$\nabla f(x_{i+1}) = f(x_0 + (i+1)h) - f(x_0 + ih) = f(x_{i+1}) - f(x_i),$$



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$$\nabla^2 f(x) = \nabla (\nabla f(x)) = \nabla (f(x) - f(x - h)) = \nabla f(x) - \nabla f(x)$$

= {f(x) - f(x - h)} - {f(x - h) - f(x - 2h)}
= f(x) - 2f(x - h) + f(x - 2h)

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Interpolation with equally spaced data The backward differences can be written in a tabular form as in Table 3

x	f(x)	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
x_0	$f(x_0)$			
		$\nabla f(x_0) =$		
		$f(x_1) - f(x_0)$		
			$\nabla^2 f(x_0) =$	
x_1	$f(x_1)$		$\nabla f(x_1) - \nabla f(x_0)$	
		$\nabla f(x_1) =$		$\nabla^3 f(x_0)$
		$f(x_2) - f(x_1)$		$\nabla^2 f(x_1) - \nabla$
			$\nabla^2 f(x_1) =$	
x_2	$f(x_2)$		$\nabla f(x_2) - \nabla f(x_1)$	< ≣> ≣ •୨ ୯
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Example

Construct the backward difference table for the data

x	-1	0	1	2
f(x)	-8	3	1	12

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Interpolation with equally spaced data Solution: We have the following difference table:

x	f(x)	abla f(x)	$\nabla^2 f(x)$	$ abla^3 f(x)$
-1	-8			
		3 - (-8) = 11		
0	3		-2 - 11 = -13	
		1 - 3 = -2		13 + 13 = 26
1	1		11 + 2 = 13	
		12 - 1 = 11		
2	12			

Table: Backward Difference Table

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Thanks !!!

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