CHAPTER

29

Helical Gears

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29.1 Introduction

A helical gear has teeth in form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears. The helixes may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission.

We have already discussed in Art. 28.4 that the helical gears may be of ***single helical type*** or ***double helical type.*** In case of single helical gears there is some axial thrust between the teeth, which is a disadvantage. In order to eliminate this axial thrust, double helical gears (*i.e.*

herringbone gears) are used. It is equivalent to two single helical gears, in which equal and opposite thrusts are provided on each gear and the resulting axial thrust is zero.

29.2 Terms used in Helical Gears

The following terms in connection with helical gears, as shown in Fig. 29.1, are important from the subject point of view.

1. *Helix angle.* It is a constant angle made by the helices with the axis of rotation.

2. *Axial pitch*. It is the distance, parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by p_c. The axial pitch may also be defined as the circular pitch in the plane of rotation or the diametral plane.

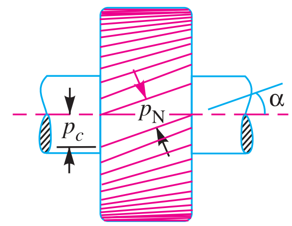

Fig. 29.1. Helical gear (nomenclature).

3. *Normal pitch.* It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by p_N. The normal pitch may also be defined as the circular pitch in the normal plane which is a plane perpendicular to the teeth. Mathematically, normal pitch,

$$p_N = p_c \cos \alpha$$

Note : If the gears are cut by standard hobs, then the pitch (or module) and the pressure angle of the hob will apply in the normal plane. On the other hand, if the gears are cut by the Fellows gear-shaper method, the pitch and pressure angle of the cutter will apply to the plane of rotation. The relation between the normal pressure angle (ϕ_N) in the normal plane and the pressure angle (ϕ) in the diametral plane (or plane of rotation) is given by

$$\tan \phi_N = \tan \phi \times \cos \alpha$$

29.3 Face Width of Helical Gears

In order to have more than one pair of teeth in contact, the tooth displacement (*i.e.* the advancement of one end of tooth over the other end) or overlap should be atleast equal to the axial pitch, such that

$$\text{Overlap} = p_c = b \tan \alpha \quad \text{...}(i)$$

The normal tooth load (W_N) has two components ; one is tangential component (W_T) and the other axial component (W_A), as shown in Fig. 29.2. The axial or end thrust is given by

$$W_A = W_N \sin \alpha = W_T \tan \alpha \quad \text{...}(ii)$$

From equation (*i*), we see that as the helix angle increases, then the tooth overlap increases. But at the same time, the end thrust as given by equation (*ii*), also increases, which is undesirable. It is usually recommended that the overlap should be 15 percent of the circular pitch.

$$\therefore \quad \text{Overlap} = b \tan \alpha = 1.15\, p_c$$

or

$$b = \frac{1.15\, p_c}{\tan \alpha} = \frac{1.15 \times \pi m}{\tan \alpha} \quad \text{... } (\because p_c = \pi m)$$

where b = Minimum face width, and

m = Module.

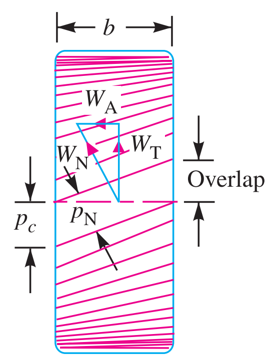

Fig. 29.2. Face width of helical gear.

Notes : 1. The maximum face width may be taken as 12.5 m to 20 m, where m is the module. In terms of pinion diameter (D_P), the face width should be 1.5 D_P to 2 D_P, although 2.5 D_P may be used.

2. In case of double helical or herringbone gears, the minimum face width is given by

$$b = \frac{2.3\, p_c}{\tan \alpha} = \frac{2.3 \times \pi m}{\tan \alpha}$$

The maximum face width ranges from 20 m to 30 m.

3. In single helical gears, the helix angle ranges from 20° to 35°, while for double helical gears, it may be made upto 45°.

29.4 Formative or Equivalent Number of Teeth for Helical Gears

The formative or equivalent number of teeth for a helical gear may be defined as the number of teeth that can be generated on the surface of a cylinder having a radius equal to the radius of curvature at a point at the tip of the minor axis of an ellipse obtained by taking a section of the gear in the normal plane. Mathematically, formative or equivalent number of teeth on a helical gear,

$$T_E = T / \cos^3 \alpha$$

where T = Actual number of teeth on a helical gear, and

α = Helix angle.

29.5 Proportions for Helical Gears

Though the proportions for helical gears are not standardised, yet the following are recommended by American Gear Manufacturer's Association (AGMA).

Pressure angle in the plane of rotation,

ϕ = 15° to 25°

Helix angle, α = 20° to 45°

Addendum = 0.8 m (Maximum)

Dedendum = 1 m (Minimum)

Minimum total depth = 1.8 m

Minimum clearance = 0.2 m

Thickness of tooth = 1.5708 m

In helical gears, the teeth are inclined to the axis of the gear.

29.6 Strength of Helical Gears

In helical gears, the contact between mating teeth is gradual, starting at one end and moving along the teeth so that at any instant the line of contact runs diagonally across the teeth. Therefore in order to find the strength of helical gears, a modified Lewis equation is used. It is given by

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y'$$

where W_T = Tangential tooth load,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y' = Tooth form factor or Lewis factor corresponding to the formative or virtual or equivalent number of teeth.

Notes : 1. The value of velocity factor (C_v) may be taken as follows :

$$C_v = \frac{6}{6+v}, \text{ for peripheral velocities from 5 m / s to 10 m / s.}$$

$$= \frac{15}{15+v}, \text{ for peripheral velocities from 10 m / s to 20 m / s.}$$

$$= \frac{0.75}{0.75+\sqrt{v}}, \text{ for peripheral velocities greater than 20 m / s.}$$

$$= \frac{0.75}{1+v} + 0.25, \text{ for non-metallic gears.}$$

2. The dynamic tooth load on the helical gears is given by

$$W_D = W_T + \frac{21\, v\, (b.C \cos^2 \alpha + W_T) \cos \alpha}{21\, v + \sqrt{b.C \cos^2 \alpha + W_T}}$$

where v, b and C have usual meanings as discussed in spur gears.

3. The static tooth load or endurance strength of the tooth is given by

$$W_S = \sigma_e.b.\pi\, m.y'$$

4. The maximum or limiting wear tooth load for helical gears is given by

$$W_w = \frac{D_P.b.Q.K}{\cos^2 \alpha}$$

where D_P, b, Q and K have usual meanings as discussed in spur gears.

In this case,
$$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

where ϕ_N = Normal pressure angle.

Example 29.1. *A pair of helical gears are to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10 000 r.p.m. and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 MPa; determine a suitable module and face width from static strength considerations and check the gears for wear, given σ_{es} = 618 MPa.*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W; $\phi = 20°$; $\alpha = 45°$; $N_P = 10\,000$ r.p.m. ; $D_P = 80$ mm $= 0.08$ m ; $D_G = 320$ mm $= 0.32$ m ; $\sigma_{OP} = \sigma_{OG} = 100$ MPa $= 100$ N/mm^2 ; $\sigma_{es} = 618$ MPa $= 618$ N/mm^2

Module and face width

Let m = Module in mm, and

b = Face width in mm.

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Since both the pinion and gear are made of the same material (*i.e.* cast steel), therefore the pinion is weaker. Thus the design will be based upon the pinion.

We know that the torque transmitted by the pinion,

$$T = \frac{P \times 60}{2\,\pi\,N_P} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 10\,000} = 14.32 \text{ N-m}$$

∴ *Tangential tooth load on the pinion,

$$W_T = \frac{T}{D_P / 2} = \frac{14.32}{0.08/2} = 358 \text{ N}$$

We know that number of teeth on the pinion,

$$T_P = D_P / m = 80 / m$$

and formative or equivalent number of teeth for the pinion,

$$T_E = \frac{T_P}{\cos^3 \alpha} = \frac{80/m}{\cos^3 45°} = \frac{80/m}{(0.707)^3} = \frac{226.4}{m}$$

∴ Tooth form factor for the pinion for 20° stub teeth,

$$y'_P = 0.175 - \frac{0.841}{T_E} = 0.175 - \frac{0.841}{226.4/m} = 0.175 - 0.0037\,m$$

We know that peripheral velocity,

$$v = \frac{\pi\,D_P . N_P}{60} = \frac{\pi \times 0.08 \times 10\,000}{60} = 42 \text{ m/s}$$

∴ Velocity factor,

$$C_v = \frac{0.75}{0.75 + \sqrt{v}} = \frac{0.75}{0.75 + \sqrt{42}} = 0.104 \qquad \text{...(}\because\ v \text{ is greater than 20 m/s)}$$

Since the maximum face width (b) for helical gears may be taken as 12.5 m to 20 m, where m is the module, therefore let us take

$$b = 12.5\,m$$

We know that the tangential tooth load (W_T),

$$358 = (\sigma_{OP} . C_v)\, b.\pi\, m.y'_P$$
$$= (100 \times 0.104)\, 12.5\,m \times \pi\,m\,(0.175 - 0.0037\,m)$$
$$= 409\,m^2\,(0.175 - 0.0037\,m) = 72\,m^2 - 1.5\,m^3$$

Solving this expression by hit and trial method, we find that

$$m = 2.3 \text{ say } 2.5 \text{ mm} \quad \textbf{Ans.}$$

and face width, $\quad b = 12.5\,m = 12.5 \times 2.5 = 31.25$ say 32 mm **Ans.**

Checking the gears for wear

We know that velocity ratio,

$$V.R. = \frac{D_G}{D_P} = \frac{320}{80} = 4$$

∴ Ratio factor,

$$Q = \frac{2 \times V.R.}{V.R. + 1} = \frac{2 \times 4}{4 + 1} = 1.6$$

We know that $\quad \tan \phi_N = \tan \phi \cos \alpha = \tan 20° \times \cos 45° = 0.2573$

∴ $\quad \phi_N = 14.4°$

* The tangential tooth load on the pinion may also be obtained by using the relation,

$$W_T = \frac{P}{v}, \text{ where } v = \frac{\pi\,D_P . N_P}{60} \text{ (in m/s)}$$

Top

The picture shows double helical gears which are also called herringbone gears.

Since both the gears are made of the same material (*i.e.* cast steel), therefore let us take

$$E_P = E_G = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$$

∴ Load stress factor,

$$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

$$= \frac{(618)^2 \sin 14.4°}{1.4}\left(\frac{1}{200 \times 10^3} + \frac{1}{200 \times 10^3}\right) = 0.678 \text{ N/mm}^2$$

We know that the maximum or limiting load for wear,

$$W_w = \frac{D_P .b.Q.K}{\cos^2 \alpha} = \frac{80 \times 32 \times 1.6 \times 0.678}{\cos^2 45°} = 5554 \text{ N}$$

Since the maximum load for wear is much more than the tangential load on the tooth, therefore the design is satisfactory from consideration of wear.

Example 29.2. *A helical cast steel gear with 30° helix angle has to transmit 35 kW at 1500 r.p.m. If the gear has 24 teeth, determine the necessary module, pitch diameter and face width for 20° full depth teeth. The static stress for cast steel may be taken as 56 MPa. The width of face may be taken as 3 times the normal pitch. What would be the end thrust on the gear? The tooth factor for 20° full depth involute gear may be taken as* $0.154 - \frac{0.912}{T_E}$, *where* T_E *represents the equivalent number of teeth.*

Solution. Given : α = 30° ; P = 35 kW = 35 × 10³ W ; N = 1500 r.p.m. ; T_G = 24 ; ϕ = 20° ; σ_o = 56 MPa = 56 N/mm² ; b = 3 × Normal pitch = 3 p_N

Module

Let m = Module in mm, and

D_G = Pitch circle diameter of the gear in mm.

We know that torque transmitted by the gear,

$$T = \frac{P \times 60}{2 \pi N} = \frac{35 \times 10^3 \times 60}{2 \pi \times 1500} = 223 \text{ N-m} = 223 \times 10^3 \text{ N-mm}$$

Formative or equivalent number of teeth,

$$T'_E = \frac{T_G}{\cos^3 \alpha} = \frac{24}{\cos^3 30°} = \frac{24}{(0.866)^3} = 37$$

∴ Tooth factor,

$$y' = 0.154 - \frac{0.912}{T_E} = 0.154 - \frac{0.912}{37} = 0.129$$

We know that the tangential tooth load,

$$W_T = \frac{T}{D_G / 2} = \frac{2\,T}{D_G} = \frac{2\,T}{m \times T_G} \qquad \text{... } (\because D_G = m.T_G)$$

$$= \frac{2 \times 223 \times 10^3}{m \times 24} = \frac{18\,600}{m} \text{ N}$$

and peripheral velocity,

$$v = \frac{\pi\, D_G\,.N}{60} = \frac{\pi\,.m.\,T_G\,.N}{60} \text{ mm/s} \qquad \text{...}(D_G \text{ and } m \text{ are in mm})$$

$$= \frac{\pi \times m \times 24 \times 1500}{60} = 1885\ m \text{ mm/s} = 1.885\ m \text{ m/s}$$

Let us take velocity factor,

$$C_v = \frac{15}{15 + v} = \frac{15}{15 + 1.885\,m}$$

We know that tangential tooth load,

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y' = (\sigma_o \times C_v)\, 3p_N \times \pi\, m \times y' \qquad \text{...}(\because b = 3\,p_N)$$

$$= (\sigma_o \times C_v)\, 3 \times p_c \cos \alpha \times \pi\, m \times y' \qquad \text{...}(\because p_N = p_c \cos \alpha)$$

$$= (\sigma_o \times C_v)\, 3\, \pi\, m \cos \alpha \times \pi\, m \times y' \qquad \text{...}(\because p_c = \pi\, m)$$

∴

$$\frac{18\,600}{m} = 56 \left(\frac{15}{15 + 1.885\,m}\right) 3\,\pi\, m \times \cos 30° \times \pi\, m \times 0.129$$

$$= \frac{2780\, m^2}{15 + 1.885\, m}$$

or $279\,000 + 35\,061\, m = 2780\, m^3$

Solving this equation by hit and trial method, we find that

m = 5.5 say 6 mm **Ans.**

Pitch diameter of the gear

We know that the pitch diameter of the gear,

$D_G = m \times T_G = 6 \times 24 = 144$ mm **Ans.**

Face width

It is given that the face width,

$$b = 3\, p_N = 3\, p_c \cos \alpha = 3 \times \pi\, m \cos \alpha$$

$$= 3 \times \pi \times 6 \cos 30° = 48.98 \text{ say } 50 \text{ mm}$$ **Ans.**

End thrust on the gear

We know that end thrust or axial load on the gear,

$$W_A = W_T \tan \alpha = \frac{18\,600}{m} \times \tan 30° = \frac{18\,600}{6} \times 0.577 = 1790 \text{ N}$$ **Ans.**

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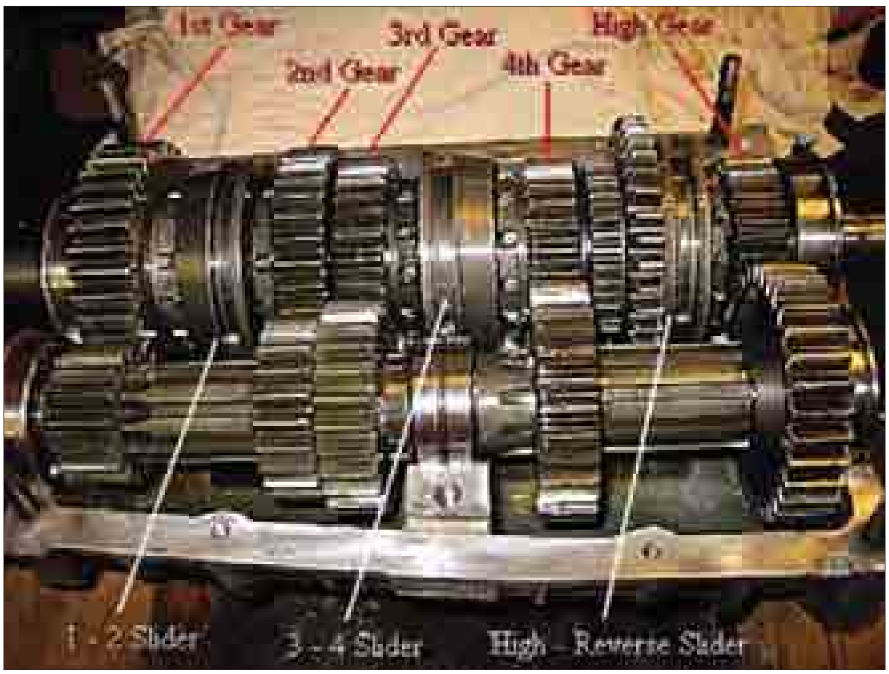
Example 29.3. *Design a pair of helical gears for transmitting 22 kW. The speed of the driver gear is 1800 r.p.m. and that of driven gear is 600 r.p.m. The helix angle is 30° and profile is corresponding to 20° full depth system. The driver gear has 24 teeth. Both the gears are made of cast steel with allowable static stress as 50 MPa. Assume the face width parallel to axis as 4 times the circular pitch and the overhang for each gear as 150 mm. The allowable shear stress for the shaft material may be taken as 50 MPa. The form factor may be taken as 0.154 – 0.912 /* T_E*, where* T_E *is the equivalent number of teeth. The velocity factor may be taken as* $\frac{350}{350 + v}$*, where v is pitch line velocity in m / min. The gears are required to be designed only against bending failure of the teeth under dynamic condition.*

Solution. Given : P = 22 kW = 22 × 10^3 W ; N_P = 1800 r.p.m.; N_G = 600 r.p.m. ; α = 30° ; ϕ = 20° ; T_P = 24 ; σ_o = 50 MPa = 50 N/mm^2 ; $b = 4\,p_c$; Overhang = 150 mm ; τ = 50 MPa = 50 N/mm^2



Gears inside a car

Design for the pinion and gear

We know that the torque transmitted by the pinion,

$$T = \frac{P \times 60}{2\,\pi\,N_P} = \frac{22 \times 10^3 \times 60}{2\,\pi \times 1800} = 116.7 \text{ N-m} = 116\,700 \text{ N-mm}$$

Since both the pinion and gear are made of the same material (*i.e.* cast steel), therefore the pinion is weaker. Thus the design will be based upon the pinion. We know that formative or equivalent number of teeth,

$$T_E = \frac{T_P}{\cos^3 \alpha} = \frac{24}{\cos^3 30°} = \frac{24}{(0.866)^3} = 37$$

∴ Form factor,
$$y' = 0.154 - \frac{0.912}{T_E} = 0.154 - \frac{0.912}{37} = 0.129$$

Top

First of all let us find the module of teeth.

Let m = Module in mm, and

D_P = Pitch circle diameter of the pinion in mm.

We know that the tangential tooth load on the pinion,

$$W_T = \frac{T}{D_P / 2} = \frac{2\,T}{D_P} = \frac{2\,T}{m \times T_P} \qquad ...(\because\ D_P = m.T_P)$$

$$= \frac{2 \times 116\ 700}{m \times 24} = \frac{9725}{m}\ \text{N}$$

and peripheral velocity, $v = \pi\, D_P.N_P = \pi\, m.T_P.N_P$

$= \pi\, m \times 24 \times 1800 = 135\ 735\ m$ mm / min $= 135.735\ m$ m / min

$$\therefore \text{ Velocity factor, } C_v = \frac{350}{350 + v} = \frac{350}{350 + 135.735\ m}$$

We also know that the tangential tooth load on the pinion,

$$W_T = (\sigma_o.C_v)\ b.\pi\ m.y' = (\sigma_o.C_v)\ 4\ p_c \times \pi\ m \times y' \qquad ...\ (\because\ b = 4\ p_c)$$

$$= (\sigma_o.C_v)\ 4 \times \pi\ m \times \pi\ m \times y' \qquad ...\ (\because\ p_c = \pi\ m)$$

$$\therefore \qquad \frac{9725}{m} = 50\left(\frac{350}{350 + 135.735\,m}\right) 4 \times \pi^2\, m^2 \times 0.129 = \frac{89\ 126\ m^2}{350 + 135.735 m}$$

$3.4 \times 10^6 + 1.32 \times 10^6\ m = 89\ 126\ m^3$

Solving this expression by hit and trial method, we find that

m = 4.75 mm say 6 mm **Ans.**

Helical gears.

We know that face width,

$b = 4\ p_c = 4\ \pi\ m = 4\ \pi \times 6 = 75.4$ say 76 mm **Ans.**

and pitch circle diameter of the pinion,

$D_P = m \times T_P = 6 \times 24 = 144$ mm **Ans.**

Since the velocity ratio is 1800 / 600 = 3, therefore number of teeth on the gear,

$T_G = 3\ T_P = 3 \times 24 = 72$

and pitch circle diameter of the gear,

$D_G = m \times T_G = 6 \times 72 = 432$ mm **Ans.**

Design for the pinion shaft

Let d_P = Diameter of the pinion shaft.

We know that the tangential load on the pinion,

$$W_T = \frac{9725}{m} = \frac{9725}{6} = 1621 \text{ N}$$

and the axial load of the pinion,

$$W_A = W_T \tan \alpha = 1621 \tan 30°$$
$$= 1621 \times 0.577 = 935 \text{ N}$$

Since the overhang for each gear is 150 mm, therefore bending moment on the pinion shaft due to the tangential load,

$$M_1 = W_T \times \text{Overhang} = 1621 \times 150 = 243\ 150 \text{ N-mm}$$

and bending moment on the pinion shaft due to the axial load,

$$M_2 = W_A \times \frac{D_P}{2} = 935 \times \frac{144}{2} = 67\,320 \text{ N-mm}$$

Since the bending moment due to the tangential load (*i.e.* M_1) and bending moment due to the axial load (*i.e.* M_2) are at right angles, therefore resultant bending moment on the pinion shaft,

$$M = \sqrt{(M_1)^2 + (M_2)^2} = \sqrt{(243\,150)^2 + (67\ 320)^2} = 252\,293 \text{ N-mm}$$

The pinion shaft is also subjected to a torque T = 116 700 N-mm, therefore equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(252\,293)^2 + (116\,700)^2} = 277\ 975 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$277\ 975 = \frac{\pi}{16} \times \tau\,(d_P)^3 = \frac{\pi}{16} \times 50\,(d_P)^3 = 9.82\,(d_P)^3$$

$\therefore$ $(d_P)^3$ = 277 975 / 9.82 = 28 307 or d_P = 30.5 say 35 mm **Ans.**

Let us now check for the principal shear stress.

We know that the shear stress induced,

$$\tau = \frac{16\,T_e}{\pi\,(d_P)^3} = \frac{16 \times 277\,975}{\pi (35)^3} = 33 \text{ N/mm}^2 = 33 \text{ MPa}$$

and direct stress due to axial load,

$$\sigma = \frac{W_A}{\frac{\pi}{4}(d_P)^2} = \frac{935}{\frac{\pi}{4}(35)^2} = 0.97 \text{ N/mm}^2 = 0.97 \text{ MPa}$$

Helical gears

∴ Principal shear stress,

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(0.97)^2 + 4(33)^2}\right] = 33 \text{ MPa}$$

Since the principal shear stress is less than the permissible shear stress of 50 MPa, therefore the design is satisfactory.

We know that the diameter of the pinion hub

$= 1.8\, d_P = 1.8 \times 35 = 63$ mm **Ans.**

and length of the hub $= 1.25\, d_P = 1.25 \times 35 = 43.75$ say 44 mm

Since the length of the hub should not be less than the face width, therefore let us take length of the hub as 76 mm. **Ans.**

Note : Since the pitch circle diameter of the pinion is 144 mm, therefore the pinion should be provided with a web. Let us take the thickness of the web as 1.8 *m*, where *m* is the module.

∴ Thickness of the web $= 1.8\, m = 1.8 \times 6 = 10.8$ say 12 mm **Ans.**

Design for the gear shaft

Let d_G = Diameter of the gear shaft.

We have already calculated that the tangential load,

$$W_T = 1621 \text{ N}$$

and the axial load, $W_A = 935$ N

∴ Bending moment due to the tangential load,

$$M_1 = W_T \times \text{Overhang} = 1621 \times 150 = 243\,150 \text{ N-mm}$$

and bending moment due to the axial load,

$$M_2 = W_A \times \frac{D_G}{2} = 935 \times \frac{432}{2} = 201\,960 \text{ N-mm}$$

∴ Resultant bending moment on the gear shaft,

$$M = \sqrt{(M_1)^2 + (M_2)^2} = \sqrt{(243\,150)^2 + (201\,960)^2} = 316\,000 \text{ N-mm}$$

Since the velocity ratio is 3, therefore the gear shaft is subjected to a torque equal to 3 times the torque on the pinion shaft.

∴ Torque on the gear shaft,

$$T = \text{Torque on the pinion shaft} \times V.R.$$

$$= 116\,700 \times 3 = 350\,100 \text{ N-mm}$$

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(316\,000)^2 + (350\,100)^2} = 472\,000 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$472\,000 = \frac{\pi}{16} \times \tau \times (d_G)^3 = \frac{\pi}{16} \times 50\,(d_G)^3 = 9.82\,(d_G)^3$$

∴ $(d_G)^3 = 472\,000 / 9.82 = 48\,065$ or $d_G = 36.3$ say 40 mm **Ans.**

Let us now check for the principal shear stress.

We know that the shear stress induced,

$$\tau = \frac{16\, T_e}{\pi\,(d_G)^3} = \frac{16 \times 472\,000}{\pi\,(40)^3} = 37.6 \text{ N/mm}^2 = 37.6 \text{ MPa}$$

and direct stress due to axial load,

$$\sigma = \frac{W_A}{\frac{\pi}{4}(d_G)^2} = \frac{935}{\frac{\pi}{4}(40)^2} = 0.744 \text{ N/mm}^2 = 0.744 \text{ MPa}$$

∴ Principal shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(0.744)^2 + 4\,(37.6)^2}\right] = 37.6 \text{ MPa}$$

Since the principal shear stress is less than the permissible shear stress of 50 MPa, therefore the design is satisfactory.

We know that the diameter of the gear hub

$= 1.8\ d_G = 1.8 \times 40 = 72$ mm **Ans.**

and length of the hub $= 1.25\ d_G = 1.25 \times 40 = 50$ mm

We shall take the length of the hub equal to the face width, *i.e.* 76 mm. **Ans.**

Since the pitch circle diameter of the gear is 432 mm, therefore the gear should be provided with four arms. The arms are designed in the similar way as discussed for spur gears.

Design for the gear arms

Let us assume that the cross-section of the arms is elliptical with major axis (a_1) equal to twice the minor axis (b_1). These dimensions refer to hub end.

∴ Section modulus of arms,

$$Z = \frac{\pi b_1 (a_1)^2}{32} = \frac{\pi\,(a_1)^3}{64} = 0.05\,(a_1)^3 \qquad \left(\because b_1 = \frac{a_1}{2}\right)$$

Since the arms are designed for the stalling load and it is taken as the design tangential load divided by the velocity factor, therefore

Stalling load,

$$W_S = \frac{W_T}{C_v} = 1621\left(\frac{350 + 135.735m}{350}\right)$$

$$= 1621\left(\frac{350 + 135.735 \times 6}{350}\right) = 5393 \text{ N}$$

∴ Maximum bending moment on each arm,

$$M = \frac{W_S}{n} \times \frac{D_G}{2} = \frac{5393}{4} \times \frac{432}{2} = 291\ 222 \text{ N-mm}$$

We know that bending stress (σ_b),

$$42 = \frac{M}{Z} = \frac{291\ 222}{0.05\,(a_1)^3} = \frac{5824 \times 10^3}{(a_1)^3} \qquad \text{... (Taking } \sigma_b = 42 \text{ N/mm}^2\text{)}$$

∴ $(a_1)^3 = 5824 \times 10^3 / 42 = 138.7 \times 10^3$ or $a_1 = 51.7$ say 54 mm **Ans.**

and $b_1 = a_1 / 2 = 54 / 2 = 27$ mm **Ans.**

Since the arms are tapered towards the rim and the taper is 1/16 mm per mm length of the arm (or radius of the gear), therefore

Major axis of the arm at the rim end,

$$a_2 = a_1 - \text{Taper} = a_1 - \frac{1}{16} \times \frac{D_G}{2}$$

$$= 54 - \frac{1}{16} \times \frac{432}{2} = 40 \text{ mm} \quad \textbf{Ans.}$$

and minor axis of the arm at the rim end,

$b_2 = a_2 / 2 = 40 / 2 = 20$ mm **Ans.**

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Design for the rim

The thickness of the rim for the pinion may be taken as 1.6 *m* to 1.9 *m*, where *m* is the module. Let us take thickness of the rim for pinion,

$$t_{RP} = 1.6\,m = 1.6 \times 6 = 9.6 \text{ say } 10 \text{ mm} \quad \textbf{Ans.}$$

The thickness of the rim for the gear (t_{RG}) is given by

$$t_{RG} = m\sqrt{\frac{T_G}{n}} = 6\sqrt{\frac{72}{4}} = 25.4 \text{ say } 26 \text{ mm} \quad \textbf{Ans.}$$

EXERCISES

1. A helical cast steel gear with 30° helix angle has to transmit 35 kW at 2000 r.p.m. If the gear has 25 teeth, find the necessary module, pitch diameters and face width for 20° full depth involute teeth. The static stress for cast steel may be taken as 100 MPa. The face width may be taken as 3 times the normal pitch. The tooth form factor is given by the expression $y' = 0.154 - 0.912/T_E$, where T_E represents the equivalent number of teeth. The velocity factor is given by $C_v = \frac{6}{6+v}$, where v is the peripheral speed of the gear in m/s.

[Ans. 6 mm ; 150 mm ; 50 mm]

2. A pair of helical gears with 30° helix angle is used to transmit 15 kW at 10 000 r.p.m. of the pinion. The velocity ratio is 4 : 1. Both the gears are to be made of hardened steel of static strength 100 N/mm^2. The gears are 20° stub and the pinion is to have 24 teeth. The face width may be taken as 14 times the module. Find the module and face width from the standpoint of strength and check the gears for wear.

[Ans. 2 mm ; 28 mm]

Gears inside a car engine.

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3. A pair of helical gears consist of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 r.p.m. The normal pressure angle is 20° while the helix angle is 25°. The face width is 40 mm and the normal module is 4 mm. The pinion as well as gear are made of steel having ultimate strength of 600 MPa and heat treated to a surface hardness of 300 B.H.N. The service factor and factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of the gears. **[Ans. 8.6 kW]**
4. A single stage helical gear reducer is to receive power from a 1440 r.p.m., 25 kW induction motor. The gear tooth profile is involute full depth with 20° normal pressure angle. The helix angle is 23°, number of teeth on pinion is 20 and the gear ratio is 3. Both the gears are made of steel with allowable beam stress of 90 MPa and hardness 250 B.H.N.

   (*a*) Design the gears for 20% overload carrying capacity from standpoint of bending strength and wear.

   (*b*) If the incremental dynamic load of 8 kN is estimated in tangential plane, what will be the safe power transmitted by the pair at the same speed?

QUESTIONS

1. What is a herringbone gear? Where they are used?
2. Explain the following terms used in helical gears :

   (*a*) Helix angle; (*b*) normal pitch; and

   (*c*) axial pitch.
3. Define formative or virtual number of teeth on a helical gear. Derive the expression used to obtain its value.
4. Write the expressions for static strength, limiting wear load and dynamic load for helical gears and explain the various terms used therein.

OBJECTIVE TYPE QUESTIONS

1. If T is the actual number of teeth on a helical gear and ϕ is the helix angle for the teeth, the formative number of teeth is written as

   (*a*) $T \sec^3 \phi$ (*b*) $T \sec^2 \phi$

   (*c*) $T/\sec^3\phi$ (*d*) $T \operatorname{cosec} \phi$
2. In helical gears, the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth, is called

   (*a*) normal pitch (*b*) axial pitch

   (*c*) diametral pitch (*d*) module
3. In helical gears, the right hand helices on one gear will mesh .... helices on the other gear.

   (*a*) right hand (*b*) left hand
4. The helix angle for single helical gears ranges from

   (*a*) 10° to 15° (*b*) 15° to 20°

   (*c*) 20° to 35° (*d*) 35° to 50°
5. The helix angle for double helical gears may be made up to

   (*a*) 45° (*b*) 60°

   (*c*) 75° (*d*) 90°

ANSWERS

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*c*) **5.** (*a*)

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