

## Lok Nayak Jai Prakash Institute of Technology Chapra, Bihar-841302

Thomson-Milne Method ...

Dr. G.K. Prajapati

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Introduction

MILNE THOMSON METHOD (TO CONSTRUCT AN ANALYTIC FUNCTION)

MILNE THOMSON METHOD (TO CONSTRUCT AN ANALYTIC FUNCTION) Mathematics-II (Complex Variable) Lecture Notes April 24, 2020

by

Dr. G.K.Prajapati Department of Applied Science and Humanities LNJPIT, Chapra, Bihar-841302

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# WORKING RULE: TO CONSTRUCT AN ANALYTIC FUNCTION BY MILNE THOMSON METHOD

**Case I.** When *u* is given **Step-1:** Find  $\frac{\partial u}{\partial x}$  and equate it to  $\phi_1(x, y)$ . **Step-2:** Find  $\frac{\partial u}{\partial y}$  and equate it to  $\phi_2(x, y)$ . **Step-3:** Replace *x* by *z* and *y* by 0 in  $\phi_1(x, y)$  to get  $\phi_1(z, 0)$ . **Step-4:** Replace *x* by *z* and *y* by 0 in  $\phi_2(x, y)$  to get  $\phi_2(z, 0)$ . **Step-5:** Find f(z) by the formula  $f(z) = \int [\phi_1(z, 0) - i\phi_2(z, 0)] dz + c$ 

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If  $u = x^2 - y^2$ , find a corresponding analytic function.

**Solution:** Here given that  $u = x^2 - y^2$ . So that  $\frac{\partial u}{\partial x} = 2x = \phi_1(x, y)$  and  $\frac{\partial u}{\partial y} = -2y = \phi_2(x, y)$ . On replacing x by z and y by 0,

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$$f(z) = \int [\phi_1(z,0) - i\phi_2(z,0)] dz + c$$
  
=  $\int (2z) dz + c$   
=  $z^2 + c$ 

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Show that  $e^x(x \cos y - y \sin y)$  is a harmonic function. Find the analytic function for which  $e^x(x \cos y - y \sin y)$  is imaginary part.

**Solution**: Here  $v = e^x(x\cos y - y\sin y)$ 

 $\frac{\partial v}{\partial x} = e^x (x \cos y - y \sin y) + e^x \cos y = \psi_2(x, y)(say), \quad (1)$ 

$$\frac{\partial v}{\partial y} = e^x (-x \sin y - y \cos y - \sin y) = \psi_1(x, y)(say), \quad (2)$$

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#### Example

Show that  $e^x(x \cos y - y \sin y)$  is a harmonic function. Find the analytic function for which  $e^x(x \cos y - y \sin y)$  is imaginary part.

**Solution**: Here  $v = e^x(x\cos y - y\sin y)$ 

$$\frac{\partial v}{\partial x} = e^x (x \cos y - y \sin y) + e^x \cos y = \psi_2(x, y)(say), \quad (1)$$

$$\frac{\partial v}{\partial y} = e^x(-x\sin y - y\cos y - \sin y) = \psi_1(x,y)(say), \quad (2)$$



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$$\frac{\partial^2 v}{\partial x^2} = e^x (x \cos y - y \sin y) + e^x \cos y + e^x \cos y$$
$$= e^x (x \cos y - y \sin y + 2 \cos y), \tag{3}$$

$$\frac{\partial^2 v}{\partial y^2} = e^x (-x\cos y + y\sin y - 2\cos y). \tag{4}$$

Adding equation (3) and (4), we have



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 $e^{x}(x\cos y - y\sin y + 2\cos y) + e^{x}(-x\cos y + y\sin y - 2\cos y) = 0$ Hence given function  $v = e^{x}(x\cos y - y\sin y)$  is harmonic function.

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$$f(z) = \int [\psi_1(z,0) + i\psi_2(z,0)] dz + c$$
  
=  $\int (0 + i(ze^z + e^z)) dz + c$   
=  $i(ze^z - e^z + e^z) + c$   
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