

## Lok Nayak Jai Prakash Institute of Technology Chapra, Bihar-841302



Dr. G.K. Prajapati LNJPIT, Chapra Complex function ...



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Introduction

FUNCTIONS OF A COMPLEX VARIABLE

NEIGHBORHO OF Z<sub>0</sub>

FUNCTIONS OF A COMPLEX VARIABLE

# **FUNCTIONS OF A COMPLEX VARIABLE:** f(z) is a function of a complex variable z and is denoted by w.

w = f(z)w = u + iv

where u and v are the real and imaginary parts of f(z). **NEIGHBORHOOD OF**  $Z_0$ :Let  $z_0$  is a point in the complex plane and let z be any positive number, then the set of points z such that

$$|z - z_0| < \epsilon$$

NEIGHBORHOOD is called  $\epsilon$ —neighbourhood of  $z_0$ . OF  $z_0$ FUNCTIONS OF A COMPLEX



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**LIMIT:** Let f(z) be a single valued function defined at all points in some neighbourhood of point  $z_0$ . Then f(z) is said to have the limit l as z approaches  $z_0$  along any path if given an arbitrary real number  $\epsilon > 0$ , however small there exists a real number  $\delta > 0$ . such that





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i.e. for every  $z \neq z_0$  in  $\delta$ -disc (dotted) of z-plane, f(z) has a value lying in the  $\epsilon$ -disc of w-plane.

In symbolic form,  $\lim_{z \to z_0} f(z) = I$ . NEIGHBORHOOD





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FUNCTIONS OF A Note: (I)  $\delta$  usually depends upon  $\epsilon$ . (II)  $z \to z_0$  implies that z approaches  $z_0$  along any path. The limits must be independent of the manner in which zpproaches  $z_0$  If we get two different limits as  $z \to z_0$  along two different paths then limits does not exist.

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### Example

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TIONS	$z \rightarrow 1-i$ $z+1$
PLEX	Solution: $(z+1)(z+3) = \lim_{z \to -3} (z+3) = (1-i) + 3 = 4 - i$
BORHO	$\lim_{z \to 1-i} \frac{1}{z+1} = \lim_{z \to 1-i} (z+3) = (1-i) + 3 = 4 - i$
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Example Show that  $\lim_{z\to 0} \frac{z}{|z|}$  does not exist. Chapra FUNCTIONS **NEIGHBORHOOD** FUNCTIONS NEIGHBORHOOD FUNCTIONS - 4 ⊒ >



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Solution:  $\lim_{z\to 0} \frac{z}{|z|} = \lim_{(x,y)\to(0,0)} \frac{x+iy}{\sqrt{x^2+y^2}}$   
Let  $y = mx$ ,  
 $= \lim_{x\to 0} \frac{x+imx}{\sqrt{x^2+(mx)^2}} = \lim_{x\to 0} \frac{1+im}{\sqrt{1+(m)^2}} = \frac{1+im}{\sqrt{1+m^2}}$   
The value of  $\frac{1+im}{\sqrt{1+m^2}}$  are different for different value of  $m$ .  
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## Example Show that $\lim_{z\to 0} \frac{z}{\overline{z}}$ does not exist. Solution: Case-1.

 $\lim_{z \to 0} \frac{z}{\overline{z}} = \lim_{(x,y) \to (0,0)} \frac{x + iy}{x - iy} = \lim_{x \to 0} \left[ \lim_{y \to 0} \frac{x + iy}{x - iy} \right] = \lim_{x \to 0} \frac{x}{\overline{x}} = 1$ Again **Case-2.**  $\lim_{z \to 0} \frac{z}{\overline{z}} = \lim_{(x,y) \to (0,0)} \frac{x + iy}{x - iy} =$   $\lim_{y \to 0} \left[ \lim_{x \to 0} \frac{x + iy}{x - iy} \right] = \lim_{y \to 0} \frac{iy}{-iy} = -1$ As  $z \to 0$  along two different paths, we get different limits.

Hence the limit does not exist.

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#### Example

Show that  $\lim_{z \to 0}^{z} does not exist.$  $z \rightarrow 0 \bar{2}$ Solution: Case-1.  $\lim_{z \to 0} \frac{z}{\bar{z}} = \lim_{(x,y) \to (0,0)} \frac{x + iy}{x - iy} = \lim_{x \to 0} \left[ \lim_{y \to 0} \frac{x + iy}{x - iy} \right] = \lim_{x \to 0} \frac{x}{x} = 1$ Again Case-2.  $\lim_{z \to 0} \frac{z}{\overline{z}} = \lim_{(x,y) \to (0,0)} \frac{x+iy}{x-iy} =$  $\lim_{y \to 0} \left[ \lim_{x \to 0} \frac{x + iy}{x - iy} \right] = \lim_{y \to 0} \frac{iy}{-iy} = -1$ As  $z \to 0$  along two different paths, we get different limits. Hence the limit does not exist.

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#### Complex function ... Dr. G.K. Prajapati Exercise Chapra Show that the limit does not exist 1. $\lim_{z \to 0} \frac{Im(z)^3}{Re(z)^3}$ 2. $\lim_{z \to 0} \frac{z}{(\bar{z})^2}$ 3. $\lim_{z \to 0} \frac{Re(z)^2}{Im(z)}$ FUNCTIONS OF A Find the limit of the following $\lim_{z \to \infty} \frac{Re(z)^2}{z}$ **Ans.** 0 6. $\lim_{z \to 1+i} \frac{2z^3}{(Im(z)^2)}$ **Ans.** 2(-1+i)**NEIGHBORHOOD** 5. $z \rightarrow 0$ 7. $\lim_{z \to 0} \frac{z^2 + 6z + 3}{z^2 + 2z + 2}$ **Ans.** 3/2. OF A NEIGHBORHOOD OF A



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**Continuity:** The function f(z) of a complex variable z is said to be continuous at the point  $z_0$  if for any given positive number  $\epsilon$ , we can find a number  $\delta$  such that  $|f(z) - f(z_0)| < \epsilon$ for all points z of the domain satisfying

$$|z - z_0| < \delta$$

A (1) < (1) < (1) </p>

NEIGHBORHOOD f(z) is said to be continuous at  $z = z_0$  if  $\lim_{z \to 0} f(z) = f(z_0)$ 



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#### Example

at z = i

#### Examine the continuity of the function

$$f(z) = \begin{cases} \frac{z^3 - iz^2 + z - i}{z - i}, & z \neq i \\ 0, & z = i \end{cases}$$

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**NEIGHBORHOOD** 

OF  $Z_0$ 

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NEIGHBORHOOD

FUNCTIONS OF A



Solution:

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	Dr. G.K. Prajapati	LNJPIT, Chapra	Complex function			
			< • • < 6	⊒→ ∢≣→ ∢≣→		
вокно	Hence $f(z)$ is	continuous at				
LEX BLE			$\sum_{i} f(z) = f(i)$			
TIONS						
BORHO	<sup><b>D</b></sup> Also, we have $f(i) = 0$ . Thus					
LEX BLE		$=\lim_{z\to z\to z}$	$\prod_{i}(z^{2}+1) \equiv 0$			
FIONS		1.	(2, 1) 0			
PIT, apra	$z \rightarrow i$ , $z \rightarrow z$	$\rightarrow i$ $z - i$ $\lim \frac{z}{z}$	$\frac{z \to i}{(z-i)}$			
	$\lim f(z) = \lim$	m $\frac{z^2 - iz^2 + z}{.}$	$\frac{z-i}{} = \lim \frac{z-i}{}$	(-i) + 1(z - i)		



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#### Solution:

$$\lim_{z \to i} f(z) = \lim_{z \to i} \frac{z^3 - iz^2 + z - i}{z - i} = \lim_{z \to i} \frac{z^2(z - i) + 1(z - i)}{z - i} = \lim_{z \to i} \frac{(z^2 + 1)(z - i)}{z - i}$$

$$= \lim_{z \to i} (z^2 + 1) = 0$$
Also, we have  $f(i) = 0$ . Thus
$$= \lim_{z \to i} f(z) = f(i)$$
RHCOD Hence  $f(z)$  is continuous at  $z = i$ .



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#### Solution:

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#### Solution:

$$\lim_{z \to i} f(z) = \lim_{z \to i} \frac{z^3 - iz^2 + z - i}{z - i} = \lim_{z \to i} \frac{z^2(z - i) + 1(z - i)}{z - i} = \lim_{z \to i} \frac{(z^2 + 1)(z - i)}{z - i}$$
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#### Solution:

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Hood Also, we have  $f(i) = 0$ . Thus
$$= \lim_{z \to i} f(z) = f(i)$$
Hood Hence  $f(z)$  is continuous at  $z = i$ .



#### Dr. G.K. Prajapati

LNJ Cha Solution:

apati	$z^{3} - iz^{2} + z - i$ $z^{2}(z - i) + 1(z - i)$
PIT,	$\lim_{z \to i} f(z) = \lim_{z \to i} \frac{z - i}{z - i} = \lim_{z \to i} \frac{z - i}{z - i} = z - i$
apra	$\lim \frac{(z^2+1)(z-i)}{z}$
uction	$z \rightarrow i$ $z - i$
TIONS	
LEX BLE	$=\lim_{z\to i}(z^2+1)=0$
IBORHO	<sup>OD</sup> Also, we have $f(i)=0$ . Thus
TIONS	$1$ $\theta$ $($ $) \theta ( t )$
LEX BLE	$=\lim_{z\to i}f(z)=f(i)$
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TIONS	
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Solution:

japati	$z^{3} - iz^{2} + z - i$ $z^{2}(z - i) + 1(z - i)$	
JPIT, Japra	$\lim_{z \to i} f(z) = \lim_{z \to i} \frac{z - iz}{z - i} = \lim_{z \to i} \frac{z - iz}{z - i} = \lim_{z \to i} \frac{z - iz}{z - i} = \lim_{z \to i} \frac{z - iz}{z - i} = \lim_{z \to i} \frac{z - iz}{z - i}$	=
	$z \rightarrow i$ $z - i$	
TIONS		
PLEX ABLE	$=\lim_{z \to i} (z^2 + 1) = 0$	
HBORHO	<sup>OD</sup> Also, we have $f(i) = 0$ . Thus	
TIONS	$1$ , $\ell(\cdot)$ , $\ell(\cdot)$	
PLEX ABLE	$=\lim_{z\to i}f(z)=f(i)$	
HBORHO	<sup>OD</sup> Hence $f(z)$ is continuous at $z = i$ .	
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#### Example

Show that the function f(z) defined by

$$f(z) = \begin{cases} \frac{Re(z)}{z}, & z \neq 0\\ 0, & z = 0 \end{cases}$$

is not continuous at z = 0.

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VARIABLE

 $\begin{array}{c} \text{NEIGHBORHO} \\ \text{OF} \ Z_0 \end{array}$ 

FUNCTIONS OF A

$$\lim_{z \to 0} f(z) = \lim_{z \to 0} \frac{Re(z)}{z} = \lim_{(x,y) \to (0,0)} \frac{x}{x+iy} = \lim_{x \to 0} \left[ \lim_{y \to 0} \frac{x}{x+iy} \right] = \lim_{x \to 0} \frac{x}{x} = 1$$

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#### NEIGHBORHCOD Solution:Here

OF Z<sub>0</sub>

OF A COMPLEX VARIABLE

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FUNCTIONS OF A

$$\lim_{z \to 0} f(z) = \lim_{z \to 0} \frac{Re(z)}{z} = \lim_{(x,y) \to (0,0)} \frac{x}{x+iy} =$$
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FUNCTIONS OF A

$$\lim_{z \to 0} f(z) = \lim_{z \to 0} \frac{Re(z)}{z} = \lim_{(x,y) \to (0,0)} \frac{x}{x + iy} =$$
$$\lim_{x \to 0} \left[ \lim_{y \to 0} \frac{x}{x + iy} \right] = \lim_{x \to 0} \frac{x}{x} = 1$$

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OF  $Z_0$ 

FUNCTIONS OF A COMPLEX VARIABLE

 $\begin{array}{l} \text{NEIGHBORHO} \\ \text{OD} \\ \text{OF} \ Z_0 \end{array}$ 

FUNCTIONS OF A

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FUNCTIONS OF A COMPLEX VARIABLE

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FUNCTIONS OF A

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### Complex function ... Dr. G.K. Prajapati Also $\lim_{z \to 0} f(z) = \lim_{z \to 0} \frac{Re(z)}{z} = \lim_{(x,y) \to (0,0)} \frac{x}{x + iy} = \lim_{y \to 0} \left[ \lim_{x \to 0} \frac{x}{x + iy} \right] = \lim_{y \to 0} \frac{0}{0 + iy} = 0$ Chapra FUNCTIONS OF A NEIGHBORHOOD FUNCTIONS OF A **NEIGHBORHOOD** OF A - 4 同下 4 日下 4 日下



### Complex function ... Dr. G.K. Prajapati Also $\lim_{z \to 0} f(z) = \lim_{z \to 0} \frac{Re(z)}{z} = \lim_{(x,y) \to (0,0)} \frac{x}{x + iy} = \lim_{y \to 0} \left[ \lim_{x \to 0} \frac{x}{x + iy} \right] = \lim_{y \to 0} \frac{0}{0 + iy} = 0$ Chapra FUNCTIONS OF A NEIGHBORHOOD FUNCTIONS OF A **NEIGHBORHOOD** OF A - 4 同下 4 日下 4 日下



### Complex function ... Dr. G.K. Prajapati Also $\lim_{z \to 0} f(z) = \lim_{z \to 0} \frac{Re(z)}{z} = \lim_{(x,y) \to (0,0)} \frac{x}{x + iy} = \lim_{y \to 0} \left[ \lim_{x \to 0} \frac{x}{x + iy} \right] = \lim_{y \to 0} \frac{0}{0 + iy} = 0$ Chapra FUNCTIONS OF A NEIGHBORHOOD FUNCTIONS OF A NEIGHBORHOOD OF A - 4 同下 4 日下 4 日下















#### Complex function ... Dr. G.K. Prajapati Also $$\begin{split} \lim_{z \to 0} f(z) &= \lim_{z \to 0} \frac{Re(z)}{z} = \lim_{(x,y) \to (0,0)} \frac{x}{x + iy} = \\ \lim_{y \to 0} \left[ \lim_{x \to 0} \frac{x}{x + iy} \right] &= \lim_{y \to 0} \frac{0}{0 + iy} = 0 \end{split}$$ Chapra FUNCTIONS OF A AS lim for two different paths, limit have two different values. **NEIGHBORHOOD** $z \rightarrow 0$ FUNCTIONS OF A NEIGHBORHOOD OF A - 4 回 ト 4 ヨ ト 4 ヨ ト Dr. G.K. Prajapati LNJPIT. Chapra Complex function ...



#### Complex function ... Dr. G.K. Prajapati Also $$\begin{split} \lim_{z \to 0} f(z) &= \lim_{z \to 0} \frac{Re(z)}{z} = \lim_{(x,y) \to (0,0)} \frac{x}{x + iy} = \\ \lim_{y \to 0} \left[ \lim_{x \to 0} \frac{x}{x + iy} \right] &= \lim_{y \to 0} \frac{0}{0 + iy} = 0 \end{split}$$ Chapra FUNCTIONS OF A AS lim for two different paths, limit have two different values. **NEIGHBORHOOD** $z \rightarrow 0$ So the limit does not exist. Thus f(z) is not continuous at FUNCTIONS z = 0.OF A **NEIGHBORHOOD** OF A ▲圖▶ ▲屋▶ ▲屋▶ Dr. G.K. Prajapati LNJPIT. Chapra Complex function ...



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#### Exercise

#### Examine the continuity of the following functions

(1.)  $f(z) = \begin{cases} \frac{Im(z)}{|z|}, z \neq 0 \\ 0, & z = 0 \end{cases}$ Ans. Not Continuous (2.)  $f(z) = \frac{z^2 + 3z + 4}{z^2 + i}$  at z = 1 - i Ans. Continuous



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FUNCTIONS OF A

### DIFFERENTIABILITY:

Let f(z) be a single valued function of the variable z, then  $f'(z)=\lim_{\delta z\to 0}\frac{f(z+\delta z)-f(z)}{\delta z}$ 

provided that the limit exists and is independent of the path along which  $\delta z \rightarrow 0$ . Let P be a fixed point and Q be a neighbouring point. The point Q may approach P along any straight line or curved path.

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Let  $f(\boldsymbol{z})$  be a single valued function of the variable  $\boldsymbol{z},$  then

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#### Example

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FUNCTIONS OF A  $\text{If } f(z) = \begin{cases} \frac{x^3y(y-ix)}{x^6+y^2}, z \neq 0 \\ 0, \qquad z=0 \end{cases} \text{ Then discuss } \frac{df}{dz} \text{ at } z=0. \end{cases}$ 

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FUNCTIONS OF A **Solution:** If  $z \to 0$  along radius vector y = mx.

$$\lim_{z \to 0} \frac{f(z) - f(0)}{z} = \lim_{z \to 0} \left[ \frac{\frac{x^3 y(y - ix)}{x^6 + y^2} - 0}{x + iy} \right] = \lim_{z \to 0} \left[ \frac{-ix^3 y(x + iy)}{(x^6 + y^2)(x + iy)} \right]$$

$$= \lim_{z \to 0} \left[ \frac{-ix^3y}{(x^6 + y^2)} \right] = \lim_{x \to 0} \left[ \frac{-ix^3(mx)}{(x^6 + m^2x^2)} \right] = \lim_{x \to 0} \left[ \frac{-imx^2}{(x^4 + m^2)} \right] = 0$$

Complex function ...

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$$= \lim_{z \to 0} \left[ \frac{-ix^3 y}{(x^6 + y^2)(x + iy)} \right] = \lim_{z \to 0} \left[ \frac{-ix^3 (mx)}{(x^6 + y^2)(x + iy)} \right] = \lim_{z \to 0} \left[ \frac{-ix^3 (mx)}{(x^6 + y^2)(x + iy)} \right]$$

$$\lim_{z \to 0} \left[ (x^6 + y^2) \right]^{-\lim_{x \to 0}} \left[ (x^6 + m^2 x^2) \right]^{-1} \\
\lim_{x \to 0} \left[ \frac{-imx^2}{(x^4 + m^2)} \right]^{-1} = 0$$

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$$\lim_{z \to 0} \frac{f(z) - f(0)}{z} = \lim_{z \to 0} \left[ \frac{\frac{x^3 y(y - ix)}{x^6 + y^2} - 0}{\frac{x^6 + y^2}{x + iy}} \right] = \lim_{z \to 0} \left[ \frac{-ix^3 y(x + iy)}{(x^6 + y^2)(x + iy)} \right]$$
$$= \lim_{z \to 0} \left[ \frac{-ix^3 y}{(x^6 + y^2)} \right] = \lim_{x \to 0} \left[ \frac{-ix^3 (mx)}{(x^6 + m^2 x^2)} \right] = \lim_{x \to 0} \left[ \frac{-imx^2}{(x^4 + m^2)} \right] = 0$$

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Complex function ...

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But along  $y = x^3$ 

$$= \lim_{z \to 0} \frac{f(z) - f(0)}{z} = \lim_{z \to 0} \left[ \frac{-ix^3(y)}{(x^6 + y^2)} \right] = \lim_{x \to 0} \left[ \frac{-ix^3(x^3)}{(x^6 + (x^3)^2)} \right] = -\frac{i}{2}$$

A (1) > A (2)



## Complex function ... Dr. G.K. Prajapati But along $y = x^3$ $=\lim_{z\to 0}\frac{f(z)-f(0)}{z}=\lim_{z\to 0}\left\lceil\frac{-ix^3(y)}{(x^6+u^2)}\right\rceil=$ Chapra FUNCTIONS OF A **NEIGHBORHOOD** FUNCTIONS OF A NFIGHBORHOOD OF A



## Complex function ... Dr. G.K. Prajapati But along $y = x^3$ $= \lim_{z \to 0} \frac{f(z) - f(0)}{z} = \lim_{z \to 0} \left[ \frac{-ix^3(y)}{(x^6 + u^2)} \right] =$ Chapra $\lim_{x \to 0} \left[ \frac{-ix^3(x^3)}{(x^6 + (x^3)^2)} \right] = -\frac{i}{2}$ FUNCTIONS OF A **NEIGHBORHOOD** FUNCTIONS OF A NFIGHBORHOOD OF A



#### Complex function ... Dr. G.K. Prajapati But along $y = x^3$ $= \lim_{z \to 0} \frac{f(z) - f(0)}{z} = \lim_{z \to 0} \left[ \frac{-ix^3(y)}{(x^6 + y^2)} \right] =$ Chapra $\lim_{x \to 0} \left[ \frac{-ix^3(x^3)}{(x^6 + (x^3)^2)} \right] = -\frac{i}{2}$ FUNCTIONS OF A **NEIGHBORHOOD** FUNCTIONS OF A **NEIGHBORHOOD** OF A LNJPIT, Chapra Dr. G.K. Prajapati Complex function ...



### Complex function ... Dr. G.K. Prajapati But along $y = x^3$ $= \lim_{z \to 0} \frac{f(z) - f(0)}{z} = \lim_{z \to 0} \left[ \frac{-ix^3(y)}{(x^6 + y^2)} \right] =$ Chapra $\lim_{x \to 0} \left[ \frac{-ix^3(x^3)}{(x^6 + (x^3)^2)} \right] = -\frac{i}{2}$ FUNCTIONS OF A **NEIGHBORHOOD** FUNCTIONS OF A **NEIGHBORHOOD** OF A Complex function ...



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FUNCTIONS OF A But along  $y = x^3$  $= \lim_{z \to 0} \frac{f(z) - f(0)}{z} = \lim_{z \to 0} \left[ \frac{-ix^3(y)}{(x^6 + y^2)} \right] =$  $\lim_{x \to 0} \left[ \frac{-ix^3(x^3)}{(x^6 + (x^3)^2)} \right] = -\frac{i}{2}$ In different paths we get different values of  $\frac{df}{dz}$  i.e. 0 and  $-\frac{i}{2}$ . In such a case, the function is not differentiable at z = 0.



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FUNCTIONS OF A COMPLEX VARIABLE But along  $y = x^3$  $= \lim_{z \to 0} \frac{f(z) - f(0)}{z} = \lim_{z \to 0} \left[ \frac{-ix^3(y)}{(x^6 + y^2)} \right] = \lim_{x \to 0} \left[ \frac{-ix^3(x^3)}{(x^6 + (x^3)^2)} \right] = -\frac{i}{2}$ 

NEIGHBORHCOD OF Z<sub>0</sub>
In different paths we get different values of  $\frac{df}{dz}$  i.e. 0 and FUNCTIONS OF A COMPLEX VARIABLE NEIGHBORHCOD OF Z<sub>0</sub>
FUNCTIONS OF Z OF A



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Example

FUNCTIONS OF A

Prove that the function  $f(z) = |z|^2$  is continuous everywhere but no where differentiable except at the origin.





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# Thanks !!!

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