

Lok Nayak Jai Prakash Institute of Technology Chapra, Bihar-841302

Classification of Partial Differential ...

> Dr. G.K. Prajapati

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Introduction

Mathematics-II (Differential Equations) Lecture Notes April 13, 2020

by

Dr. G.K.Prajapati Department of Applied Science and Humanities LNJPIT, Chapra, Bihar-841302



Classification of Partial Differential ...

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Introduction

Classification of second order partial differential equations

Consider a general second order partial differential equation for a function of two variables x and y in the form

$$Rr + Ss + Tt + f(x, y, z, p, q) = 0,$$

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$. Also R, S and T are continuous functions of x and y only possessing partial derivatives defined in a domain D on the (x - y) plan.



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Then the given equation is said to be

- Hyperbolic at a point (x, y) in domain D if $S^2 4RT > 0$
- Parabolic at a point (x, y) in domain D if $S^2 4RT = 0$

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Example

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Classify the following partial differential equation

1.
$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}.$$

2.
$$2\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = 2.$$

3.
$$(xy - 1)r - 2(x^2y^2 - 1)s - (xy + 1)t + xp + yq = 0$$

Image: A matrix and a matrix

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Solution (1.) The given equation can be written as r - t = 0. Comparing the given equation with

Rr + Ss + Tt + f(x, y, z, p, q) = 0,

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Solution (2.) The given equation can be written as 2r + s + 3t - 2 = 0. Comparing the given equation with

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Solution (3.) Comparing the given equation with Rr + Ss + Tt + f(x, y, z, p, q) = 0, We have $R = (xy - 1), S = -2(x^2y^2 - 1)$ and T = -(x - 1).

 $S^{2} - 4RT = (-2(x^{2}y^{2} - 1))^{2} - 4.((xy - 1)).(-(xy + 1)) \implies 4(x^{2}y^{2} - 1))^{2} + 4.((x^{2}y^{2} - 1)).$

$$S^2 - 4RT = 4x^2y^2(x^2y^2 - 1).$$



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$$\begin{split} S^2 - 4RT &= (-2(x^2y^2 - 1))^2 - 4.((xy - 1)).(-(xy + 1)) \implies \\ &\quad 4(x^2y^2 - 1))^2 + 4.((x^2y^2 - 1)). \end{split}$$

$$S^2 - 4RT = 4x^2y^2(x^2y^2 - 1).$$



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Case-1: Either x = 0 or y = 0 or both x = y = 0. In this case $S^2 - 4RT = 0$, hence given equation is parabola.

Case-2: If $xy = \pm 1$, then in this case $S^2 - 4RT = 0$, hence given equation is parabola.

Case-3: If $x^2y^2 > 1$, then in this case $S^2 - 4RT > 0$, hence given equation is hyperbola.

Case-4: If $x^2y^2 < 1$, then in this case $S^2 - 4RT < 0$, hence given equation is elliptic.



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Example ((AKU-CE-II,2019))

Classify the partial differential equation $\frac{\partial^2 u}{\partial t^2} + t \frac{\partial^2 u}{\partial x \partial t} + x \frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + 6u = 0.$

Solution: Comparing the given equation with

$$Rr + Ss + Tt + f(x, y, z, p, q) = 0,$$

We have R = x, S = t and T = 1. Put these values in

 $S^2 - 4RT = (t)^2 - 4.(x).(1) \implies S^2 - 4RT = t^2 - 4x.$

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Case-1: If $x = t^2/4$, then in this case $S^2 - 4RT = 0$, hence given equation is parabola.

Case-2: If $x < t^2/4$, then in this case $S^2 - 4RT > 0$, hence given equation is hyperbola.

Case-3: If $x > t^2/4$, then in this case $S^2 - 4RT < 0$, hence given equation is elliptic.



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Example ((AKU-CE-II,2019))

The region in which the following partial differential equation $x^3 \frac{\partial^2 u}{\partial x^2} + 3 \frac{\partial^2 u}{\partial x \partial y} + 27 \frac{\partial^2 u}{\partial y^2} + 5u = 0.$

Solution: Comparing the given equation with

$$Rr + Ss + Tt + f(x, y, z, p, q) = 0,$$

We have $R=x^3$, S=3 and T=27.Put these values in

 $S^{2} - 4RT = (3)^{2} - 4.(x^{3}).(27) \implies S^{2} - 4RT = 9 - 108x^{3}.$

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Case-1: If $x = (1/12)^{1/3}$, then in this case



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Classify the following PDE:

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(1.)
$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

(2.) $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0.$
(3.) $xyr - (x^2 - y^2)s - xyt + py - qx = 2(x^2 - y^2)$
(4.) $x^2(y - 1)r - x(y^2 - 1)s + y(y - 1)t + xyp - q = 0$

Exercise

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