

# <span id="page-0-0"></span>Lok Nayak Jai Prakash Institute of Technology Chapra, Bihar-841302

[Homogeneous](#page-16-0) Linear Partial Differential ...

> Dr. G.K. **Prajapati**

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Mathematics-II (Differential Equations) Lecture Notes April 10, 2020

by

Dr. G.K.Prajapati Department of Applied Science and Humanities LNJPIT, Chapra, Bihar-841302

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# Working rule for finding Particular Integral P.I. of reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients be  $F(D, D')z = \phi(x, y)$ **Case-I**: When  $\phi(x, y) = e^{ax + by}$  and  $F(a, b) \neq 0$ . Then, we get the P.I. by replacing D by a and  $D'$  by b. i.e.

$$
P.I. = \frac{1}{F(D, D')}e^{ax + by} = \frac{1}{F(a, b)}e^{ax + by}
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### Example

Solve the PDE 
$$
(DD' + aD + bD' + ab)z = e^{mx+ny}
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**Solution:** The given equation can be written  $\mathsf{as}(D+b)(D'+a)z = e^{mx+ny},$ which is reducible. Hence complementary function (C.F.) is

 $C.F. = e^{-bx} f_1(y) + e^{-ay} f_2(x)$ ,  $f_1$  and  $f_2$  are arbitrary constant.

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P.I. = \frac{1}{F(D, D')}{e^{ax+by}} = \frac{1}{(D+b)(D'+a)}{e^{mx+ny}} = \frac{1}{(m+b, n+a)}{e^{mx+ny}}.
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#### Example

# Solve the PDE  $(D^2 - D'^2 + D - D')z = e^{2x+3y}$ .

**Solution:** The given equation can be written as

$$
[(D - D')(D + D') + D - D'] z = e^{2x+3y} \implies
$$
  

$$
(D - D')(D + D' + 1)z = e^{2x+3y},
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 $P.I. = \frac{1}{P(D)}$  $\frac{1}{F(D,D')}e^{{\bm{a}}{\bm{x}}+{\bm{b}}{\bm{y}}} = \frac{1}{(D-D')(D+D'+1)}e^{2x+3y}=$  $\frac{1}{(2-3)(2+3+1)}e^{2x+3y} \implies -\frac{1}{6}$  $\frac{1}{6}e^{2x+3y}$ .

Hence the required solution is

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z = f_1(y+x) + e^{-x} f_2(y-x) - \frac{1}{6} e^{2x+3y}.
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 $\mathcal{A} \cap \overline{\mathcal{A}} \longrightarrow \mathcal{A} \subseteq \mathcal{A}$ 

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#### <span id="page-27-0"></span>Example

Solve the PDE  $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$ .

**Solution:** The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$
z = \sum Ae^{hx+ky}.
$$

Therefore we have  $Dz=\sum Ahe^{hx+ky}$  ,  $D^2z=\sum Ah^2e^{hx+ky}$  and  $DD'z = \sum Ahke^{hx+ky}$ . Put all these values in given equation  $(D^2 - 4DD' + D - 1)z = 0$ , we have  $\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} =$ 0.

 $\implies \sum A(h^2 - 4hk + h - 1)e_{n+1}^{hx+ky} = 0$  $\implies \sum A(h^2 - 4hk + h - 1)e_{n+1}^{hx+ky} = 0$  $\implies \sum A(h^2 - 4hk + h - 1)e_{n+1}^{hx+ky} = 0$  $\implies \sum A(h^2 - 4hk + h - 1)e_{n+1}^{hx+ky} = 0$  $\implies \sum A(h^2 - 4hk + h - 1)e_{n+1}^{hx+ky} = 0$  $\implies \sum A(h^2 - 4hk + h - 1)e_{n+1}^{hx+ky} = 0$  $\implies \sum A(h^2 - 4hk + h - 1)e_{n+1}^{hx+ky} = 0$  $\implies \sum A(h^2 - 4hk + h - 1)e_{n+1}^{hx+ky} = 0$  $\implies \sum A(h^2 - 4hk + h - 1)e_{n+1}^{hx+ky} = 0$ 

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**Solution:** The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$
z = \sum Ae^{hx+ky}.
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Therefore we have  $Dz=\sum Ahe^{hx+ky}$  ,  $D^2z=\sum Ah^2e^{hx+ky}$  and  $DD'z = \sum Ahke^{hx+ky}$ . Put all these values in given equation  $(D^{2} – 4DD' + D – 1)z = 0$ , we have  $\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} =$ 0.

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#### <span id="page-30-0"></span>Example

Solve the PDE 
$$
(D^2 - 4DD' + D - 1)z = e^{3x-2y}
$$
.

**Solution:** The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$
z = \sum Ae^{hx+ky}.
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Therefore we have  $Dz=\sum Ahe^{\bm h\bm x+\bm k\bm y},$   $D^2z=\sum Ah^2e^{hx+ky}$  and  $DD'z = \sum Ahke^{hx+ky}$ . Put all these values in given equation  $(D^{2} – 4DD' + D – 1)z = 0$ , we have  $\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} =$ 0.

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#### <span id="page-31-0"></span>Example

Solve the PDE 
$$
(D^2 - 4DD' + D - 1)z = e^{3x-2y}
$$
.

**Solution:** The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

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z = \sum Ae^{hx+ky}.
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Therefore we have  $Dz=\sum Ahe^{hx+ky}$ , $D^2z=\sum A h^2 e^{hx+ky}$ and  $DD'z = \sum Ahke^{hx+ky}$ . Put all these values in given equation  $(D^{2} – 4DD' + D – 1)z = 0$ , we have  $\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} =$ 0.

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#### <span id="page-32-0"></span>Example

Solve the PDE 
$$
(D^2 - 4DD' + D - 1)z = e^{3x-2y}
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**Solution:** The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

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#### <span id="page-33-0"></span>Example

Solve the PDE 
$$
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#### <span id="page-34-0"></span>Example

Solve the PDE 
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**Solution:** The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

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$$
\implies \textstyle \sum A(h^2-4hk+h-1)e^{hx+ky}_{\text{max}}=0 \implies \textstyle \sum_{k=0}^{\infty} \textstyle \sum_{
$$

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Solve the PDE 
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(D^2 - 4DD' + D - 1)z = e^{3x-2y}
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$$
\implies \sum A(h^2-4hk+h-1)e^{hx+ky}_{\circlearrowright}=\mathbb{Q} \implies \lim_{r\to\infty} \sum_{k=0}^{\infty} \mathbb{Q} \text{ for } k\to\infty.
$$

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$$
\implies k = \frac{(h^2 + h - 1)}{4h}.
$$

Thus 
$$
C.F. = \sum Ae^{hx+ky}
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, where  $k = \frac{(h^2 + h - 1)}{4h}$ 

$$
P.I. = \frac{1}{F(D, D')}e^{ax+by} = \frac{1}{(D^2 - 4DD' + D - 1)}e^{3x - 2y} = \frac{1}{(3^2 - 4.3.(-2) + 3 - 1)}e^{3x - 2y} = \frac{1}{35}e^{3x - 2y}.
$$

Hence the required solution is

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z = \sum Ae^{hx+ky} + \frac{1}{35}e^{3x-2y}, \text{where } k = \frac{(h^2 + h - 1)}{4h}.
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$$
\implies k = \frac{(h^2 + h - 1)}{4h}.
$$
  
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# Exercise

Solve the following PDE:

(1)  $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$  Ans.  $z = e^x f_1(y+x) + e^{2x} f_2(y+x) + (1/2)e^{2x-1}.$ (2)  $(D^3 - 3DD' + D + 1)z = e^{2x + 3y}$  $Ans.$  $z = \sum Ae^{hx+ky} - \frac{1}{5}$  $\frac{1}{7}e^{2x+3y}$ , where  $k = \frac{(h^3 + h + 1)}{3h}$  $\frac{1}{3h}$ . (3)  $(D^2 - D'^2 - 3D')z = e^{x+2y}$  Ans.  $z = \sum Ae^{hx+ky} - \frac{1}{2}$  $\frac{1}{9}e^{x+2y}$ , where  $h =$  $k^2 + 3k$ . (4)  $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y}$  Ans.  $z = e^{-2x} f_1(y+x) + e^x f_2(y-x) - (1/4)e^{x-y}.$ 

 $4.51 \times 4.71 \times 1.71 \times$ 



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# Exercise

Solve the following PDE:

(1)  $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$  Ans.  $z = e^x f_1(y+x) + e^{2x} f_2(y+x) + (1/2)e^{2x-1}.$ (2)  $(D^3 - 3DD' + D + 1)z = e^{2x+3y}$  Ans.  $z = \sum Ae^{hx+ky} - \frac{1}{7}$  $\frac{1}{7}e^{2x+3y}$ , where  $k = \frac{(h^3 + h + 1)}{3h}$  $\frac{n+1}{3h}$ . (3)  $(D^2 - D'^2 - 3D')z = e^{x+2y}$  Ans.  $z = \sum Ae^{hx+ky} - \frac{1}{2}$  $\frac{1}{9}e^{x+2y}$ , where  $h =$  $k^2 + 3k$ . (4)  $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y}$  Ans.  $z = e^{-2x} f_1(y+x) + e^x f_2(y-x) - (1/4)e^{x-y}.$ 

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# Exercise

Solve the following PDE:

(1)  $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$  Ans.  $z = e^x f_1(y+x) + e^{2x} f_2(y+x) + (1/2)e^{2x-1}.$ (2)  $(D^3 - 3DD' + D + 1)z = e^{2x+3y}$  Ans.  $\frac{1}{7}e^{2x+3y}$ , where  $k = \frac{(h^3 + h + 1)}{3h}$  $z = \sum Ae^{hx+ky} - \frac{1}{7}$  $\frac{n+1}{3h}$ . (3)  $(D^2 - D'^2 - 3D')z = e^{x+2y}$  Ans. √  $z = \sum Ae^{hx+ky} - \frac{1}{2}$  $\frac{1}{9}e^{x+2y}$ , where  $h =$  $\overline{k^2+3k}$ . (4)  $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y}$  Ans.  $z = e^{-2x} f_1(y+x) + e^x f_2(y-x) - (1/4)e^{x-y}.$  $4.51 \times 4.71 \times 1.71 \times$ 

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### [Homogeneous](#page-0-0) Linear Partial Differential ... Dr. G.K. **Prajapati** LNJPIT, Chapra [Introduction](#page-1-0) Now  $P.I. = \frac{1}{P(D)}$  $\frac{1}{F(D,D')}\sin(x+2y)=$  $\frac{1}{(D^2 + DD' + D' - 1)} \sin(x + 2y) =$ 1  $\frac{1}{(-1^2 + (-1.2) + D' - 1)} \sin(x + 2y) = \frac{1}{D'}$  $\frac{1}{D'-4}\sin(x+2y).$  $P.I. = (D' + 4) \frac{1}{D'^2 - 4^2} \sin(x + 2y) \implies$

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Hence the required solution is

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Solve the PDE 
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(D - D^2)z = \cos(x - 3y)
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**Solution:** The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

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z = \sum Ae^{hx+ky}.
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So that  $Dz=\sum Ahe^{hx+ky}$ and  $D'^2z=\sum Ak^2e^{hx+ky}.$ By Putting these values in given equation  $(D - D'^2)z = 0$ , we have  $\sum A h e^{hx+ky} - \sum Ak^2 e^{hx+ky} = 0 \implies \sum A (h-k^2) e^{hx+ky} = 0.$ 

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# Example

Solve the PDE 
$$
(D - D^2)z = cos(x - 3y)
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**Solution:** The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

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# Exercise

Solve the following PDE:

(1) 
$$
\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x + 2y)
$$
 Ans.  
\n
$$
z = e^x f_1(y) + e^{-x} f_2(y + x) + (1/2) \sin(x + 2y).
$$
  
\n(2) 
$$
(D^2 - DD' - 2D)z = \sin(3x + 4y)
$$
 Ans.  $z = f_1(y) + e^{2x} f_2(y + x) + (1/15) [\sin(3x + 4y) + 2 \cos(3x + 4y)].$   
\n(3) 
$$
(D - D' - 1)(D - D' - 2)z = \sin(2x + 3y)
$$
 Ans.  
\n
$$
z = e^x f_1(y + x) + e^{2x} f_2(y + x) +
$$
  
\n(1/10)  $[\sin(2x + 3y) - 12 \cos(2x + 3y)].$   
\n(4) 
$$
(D^2 - D')z = \cos(3x - y)
$$
 Ans.  
\n
$$
z = \sum Ae^{hx + h^2 y} - \frac{1}{82} [-\sin(3x - y) + 9 \cos(3x - y)].
$$

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