Mathematics-II (Differential Equations)
Lecture Notes
April 10, 2020

by

Dr. G.K.Prajapati
Department of Applied Science and Humanities
LNJPIT, Chapra, Bihar-841302

Working rule for finding Particular Integral P.I. of reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients be $F(D, D')z = \phi(x, y)$

Case-I: When $\phi(x, y) = e^{ax+by}$ and $F(a, b) \neq 0$.

Then, we get the P.I. by replacing D by a and D' by b. i.e.

$$P.I. = \frac{1}{F(D, D')}e^{ax+by} = \frac{1}{F(a, b)}e^{ax+by}$$

Introduction

Working rule for finding Particular Integral P.I. of reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients be $F(D, D')z = \phi(x, y)$

Case-I: When $\phi(x, y) = e^{ax+by}$ and $F(a, b) \neq 0$.

Then, we get the P.I. by replacing D by a and D' by b. i.e.

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{F(a, b)} e^{ax+by}$$

Working rule for finding Particular Integral P.I. of reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients be
$F(D, D')z = \phi(x, y)$
Case-I: When $\phi(x, y) = e^{ax+by}$ and $F(a, b) \neq 0$.
Then, we get the P.I. by replacing D by a and D' by b. i.e.

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{F(a, b)} e^{ax+by}$$

Introduction

Working rule for finding Particular Integral P.I. of reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients be
$F(D, D')z = \phi(x, y)$
Case-I: When $\phi(x, y) = e^{ax+by}$ and $F(a, b) \neq 0$.
Then, we get the P.I. by replacing D by a and D' by b. i.e.

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{F(a, b)} e^{ax+by}$$

Introduction

Working rule for finding Particular Integral P.I. of reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given reducible/irreducible non-homogeneous linear partial differential equations with constants coefficients be $F(D, D')z = \phi(x, y)$
Case-I: When $\phi(x, y) = e^{ax+by}$ and $F(a, b) \neq 0$.
Then, we get the P.I. by replacing D by a and D' by b. i.e.

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{F(a, b)} e^{ax+by}$$

Example

Solve the PDE $(DD' + aD + bD' + ab)z = e^{mx+ny}$.

Solution: The given equation can be written as$(D+b)(D'+a)z = e^{mx+ny}$,which is reducible. Hence complementary function (C.F.) is

$$C.F. = e^{-bx} f_1(y) + e^{-ay} f_2(x),\ f_1 \text{ and } f_2 \text{ are arbitrary constant.}$$

and

$$P.I. = \frac{1}{F(D,D')} e^{ax+by} = \frac{1}{(D+b)(D'+a)} e^{mx+ny} = \frac{1}{(m+b, n+a)} e^{mx+ny}.$$

Example

Solve the PDE $(DD' + aD + bD' + ab)z = e^{mx+ny}$.

Solution: The given equation can be written as $(D+b)(D'+a)z = e^{mx+ny}$, which is reducible. Hence complementary function (C.F.) is

$$C.F. = e^{-bx} f_1(y) + e^{-ay} f_2(x),$$ f_1 and f_2 are arbitrary constant.

and

$$P.I. = \frac{1}{F(D,D')} e^{ax+by} = \frac{1}{(D+b)(D'+a)} e^{mx+ny} = \frac{1}{(m+b,n+a)} e^{mx+ny}.$$

Example

Solve the PDE $(DD' + aD + bD' + ab)z = e^{mx+ny}$.

Solution: The given equation can be written as$(D+b)(D'+a)z = e^{mx+ny}$,which is reducible. Hence complementary function (C.F.) is

$$C.F. = e^{-bx}f_1(y) + e^{-ay}f_2(x),$$ f_1 and f_2 are arbitrary constant.

and

$$P.I. = \frac{1}{F(D,D')}e^{ax+by} = \frac{1}{(D+b)(D'+a)}e^{mx+ny} = \frac{1}{(m+b, n+a)}e^{mx+ny}.$$

Example

Solve the PDE $(DD' + aD + bD' + ab)z = e^{mx+ny}$.

Solution: The given equation can be written as$(D+b)(D'+a)z = e^{mx+ny}$,which is reducible. Hence complementary function (C.F.) is

$C.F. = e^{-bx}f_1(y) + e^{-ay}f_2(x)$, f_1 and f_2 are arbitrary constant.

and

$$P.I. = \frac{1}{F(D,D')}e^{ax+by} = \frac{1}{(D+b)(D'+a)}e^{mx+ny} = \frac{1}{(m+b,n+a)}e^{mx+ny}.$$

Example

Solve the PDE $(DD' + aD + bD' + ab)z = e^{mx+ny}$.

Solution: The given equation can be written as$(D+b)(D'+a)z = e^{mx+ny}$,which is reducible. Hence complementary function (C.F.) is

$$C.F. = e^{-bx}f_1(y) + e^{-ay}f_2(x),\ f_1 \text{ and } f_2 \text{ are arbitrary constant.}$$

and

$$P.I. = \frac{1}{F(D,D')}e^{ax+by} = \frac{1}{(D+b)(D'+a)}e^{mx+ny} = \frac{1}{(m+b,n+a)}e^{mx+ny}.$$

Example

Solve the PDE $(DD' + aD + bD' + ab)z = e^{mx+ny}$.

Solution: The given equation can be written as$(D + b)(D' + a)z = e^{mx+ny}$,which is reducible. Hence complementary function (C.F.) is

$$C.F. = e^{-bx} f_1(y) + e^{-ay} f_2(x),\ f_1 \text{ and } f_2 \text{ are arbitrary constant.}$$

and

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{(D + b)(D' + a)} e^{mx+ny} = \frac{1}{(m + b, n + a)} e^{mx+ny}.$$

Example

Solve the PDE $(DD' + aD + bD' + ab)z = e^{mx+ny}$.

Solution: The given equation can be written as$(D+b)(D'+a)z = e^{mx+ny}$,which is reducible. Hence complementary function (C.F.) is

$$C.F. = e^{-bx} f_1(y) + e^{-ay} f_2(x),\ f_1 \text{ and } f_2 \text{ are arbitrary constant.}$$

and

$$P.I. = \frac{1}{F(D,D')} e^{ax+by} = \frac{1}{(D+b)(D'+a)} e^{mx+ny} = \frac{1}{(m+b, n+a)} e^{mx+ny}.$$

Example

Solve the PDE $(DD' + aD + bD' + ab)z = e^{mx+ny}$.

Solution: The given equation can be written as$(D+b)(D'+a)z = e^{mx+ny}$,which is reducible. Hence complementary function (C.F.) is

$$C.F. = e^{-bx} f_1(y) + e^{-ay} f_2(x),\ f_1 \text{ and } f_2 \text{ are arbitrary constant.}$$

and

$$P.I. = \frac{1}{F(D,D')} e^{ax+by} = \frac{1}{(D+b)(D'+a)} e^{mx+ny} = \frac{1}{(m+b,n+a)} e^{mx+ny}.$$

Hence the required solution is

$$z = e^{-bx} f_1(y) + e^{-ay} f_2(x) + \frac{1}{(m+b, n+a)} e^{mx+ny}.$$

Hence the required solution is

$$z = e^{-bx} f_1(y) + e^{-ay} f_2(x) + \frac{1}{(m+b, n+a)} e^{mx+ny}.$$

Example

Solve the PDE $(D^2 - D'^2 + D - D')z = e^{2x+3y}$.

Solution: The given equation can be written as

$$[(D-D')(D+D') + D - D']\, z = e^{2x+3y} \implies$$
$$(D-D')(D+D'+1)z = e^{2x+3y},$$

which is reducible. Hence it's complementary function (C.F.) is

$C.F. = f_1(y+x) + e^{-x} f_2(y-x)$, f_1 and f_2 are arbitrary constant.

Example

Solve the PDE $(D^2 - D'^2 + D - D')z = e^{2x+3y}$.

Solution: The given equation can be written as

$$[(D - D')(D + D') + D - D']\, z = e^{2x+3y} \implies$$
$$(D - D')(D + D' + 1)z = e^{2x+3y},$$

which is reducible. Hence it's complementary function (C.F.) is

$C.F. = f_1(y + x) + e^{-x} f_2(y - x)$, f_1 and f_2 are arbitrary constant.

Example

Solve the PDE $(D^2 - D'^2 + D - D')z = e^{2x+3y}$.

Solution: The given equation can be written as

$$[(D - D')(D + D') + D - D']\, z = e^{2x+3y} \implies (D - D')(D + D' + 1)z = e^{2x+3y},$$

which is reducible. Hence it's complementary function (C.F.) is

$C.F. = f_1(y + x) + e^{-x} f_2(y - x)$, f_1 and f_2 are arbitrary constant.

Example

Solve the PDE $(D^2 - D'^2 + D - D')z = e^{2x+3y}$.

Solution: The given equation can be written as

$$[(D - D')(D + D') + D - D']\, z = e^{2x+3y} \implies$$
$$(D - D')(D + D' + 1)z = e^{2x+3y},$$

which is reducible. Hence it's complementary function (C.F.) is

$C.F. = f_1(y + x) + e^{-x} f_2(y - x)$, f_1 and f_2 are arbitrary constant.

Example

Solve the PDE $(D^2 - D'^2 + D - D')z = e^{2x+3y}$.

Solution: The given equation can be written as

$$[(D - D')(D + D') + D - D']\, z = e^{2x+3y} \implies$$
$$(D - D')(D + D' + 1)z = e^{2x+3y},$$

which is reducible. Hence it's complementary function (C.F.) is

$$C.F. = f_1(y + x) + e^{-x} f_2(y - x),\ f_1 \text{ and } f_2 \text{ are arbitrary constant.}$$

and

$$P.I. = \frac{1}{F(D,D')}e^{ax+by} = \frac{1}{(D-D')(D+D'+1)}e^{2x+3y} = \frac{1}{(2-3)(2+3+1)}e^{2x+3y} \implies = -\frac{1}{6}e^{2x+3y}.$$

Hence the required solution is

$$z = f_1(y+x) + e^{-x}f_2(y-x) - \frac{1}{6}e^{2x+3y}.$$

and

$$P.I. = \frac{1}{F(D,D')}e^{ax+by} = \frac{1}{(D-D')(D+D'+1)}e^{2x+3y} = \frac{1}{(2-3)(2+3+1)}e^{2x+3y} \implies = -\frac{1}{6}e^{2x+3y}.$$

Hence the required solution is

$$z = f_1(y+x) + e^{-x}f_2(y-x) - \frac{1}{6}e^{2x+3y}.$$

and

$$P.I. = \frac{1}{F(D,D')}e^{ax+by} = \frac{1}{(D-D')(D+D'+1)}e^{2x+3y} = \frac{1}{(2-3)(2+3+1)}e^{2x+3y} \implies = -\frac{1}{6}e^{2x+3y}.$$

Hence the required solution is

$$z = f_1(y+x) + e^{-x}f_2(y-x) - \frac{1}{6}e^{2x+3y}.$$

and

$$P.I. = \frac{1}{F(D,D')}e^{ax+by} = \frac{1}{(D-D')(D+D'+1)}e^{2x+3y} = \frac{1}{(2-3)(2+3+1)}e^{2x+3y} \implies = -\frac{1}{6}e^{2x+3y}.$$

Hence the required solution is

$$z = f_1(y+x) + e^{-x}f_2(y-x) - \frac{1}{6}e^{2x+3y}.$$

and

$$P.I. = \frac{1}{F(D,D')}e^{ax+by} = \frac{1}{(D-D')(D+D'+1)}e^{2x+3y} = \frac{1}{(2-3)(2+3+1)}e^{2x+3y} \implies = -\frac{1}{6}e^{2x+3y}.$$

Hence the required solution is

$$z = f_1(y+x) + e^{-x}f_2(y-x) - \frac{1}{6}e^{2x+3y}.$$

and

$$P.I. = \frac{1}{F(D, D')}e^{ax+by} = \frac{1}{(D - D')(D + D' + 1)}e^{2x+3y} = \frac{1}{(2-3)(2+3+1)}e^{2x+3y} \implies = -\frac{1}{6}e^{2x+3y}.$$

Hence the required solution is

$$z = f_1(y + x) + e^{-x} f_2(y - x) - \frac{1}{6}e^{2x+3y}.$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have
$Dz = \sum Ahe^{hx+ky}$, $D^2z = \sum Ah^2e^{hx+ky}$ and
$DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation
$(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0 \implies$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have

$Dz = \sum Ahe^{hx+ky}$, $D^2z = \sum Ah^2e^{hx+ky}$ and $DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation $(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0 \implies$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have

$Dz = \sum Ahe^{hx+ky}$, $D^2z = \sum Ah^2e^{hx+ky}$ and $DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation $(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0 \implies$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have

$Dz = \sum Ahe^{hx+ky}$, $D^2z = \sum Ah^2e^{hx+ky}$ and $DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation $(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0 \implies$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have
$Dz = \sum Ahe^{hx+ky}$, $D^2z = \sum Ah^2e^{hx+ky}$ and $DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation $(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0 \implies$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have

$Dz = \sum Ahe^{hx+ky}$, $D^2z = \sum Ah^2e^{hx+ky}$ and $DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation $(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0 \implies$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have
$Dz = \sum Ahe^{hx+ky}$,$D^2z = \sum Ah^2e^{hx+ky}$and
$DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation
$(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0 \implies$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have

$Dz = \sum Ahe^{hx+ky}$,$D^2z = \sum Ah^2e^{hx+ky}$and

$DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation

$(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0 \implies$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have
$Dz = \sum Ahe^{hx+ky}$,$D^2z = \sum Ah^2e^{hx+ky}$and
$DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation
$(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0$$

Example

Solve the PDE $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

Therefore we have
$Dz = \sum Ahe^{hx+ky}$, $D^2z = \sum Ah^2e^{hx+ky}$ and
$DD'z = \sum Ahke^{hx+ky}$. Put all these values in given equation
$(D^2 - 4DD' + D - 1)z = 0$, we have

$$\sum Ah^2e^{hx+ky} - 4\sum Ahke^{hx+ky} + \sum Ahe^{hx+ky} - \sum Ae^{hx+ky} = 0.$$

$$\implies \sum A(h^2 - 4hk + h - 1)e^{hx+ky} = 0 \implies$$

$$\implies k = \frac{(h^2 + h - 1)}{4h}.$$

Thus $C.F. = \sum Ae^{hx+ky}$, where $k = \dfrac{(h^2 + h - 1)}{4h}$

$$P.I. = \frac{1}{F(D, D')}e^{ax+by} = \frac{1}{(D^2 - 4DD' + D - 1)}e^{3x-2y} = \frac{1}{(3^2 - 4.3.(-2) + 3 - 1)}e^{3x-2y} = \frac{1}{35}e^{3x-2y}.$$

Hence the required solution is

$$z = \sum Ae^{hx+ky} + \frac{1}{35}e^{3x-2y}, \text{where } k = \frac{(h^2 + h - 1)}{4h}.$$

$$\Longrightarrow\ k = \frac{(h^2 + h - 1)}{4h}.$$

Thus $C.F. = \sum Ae^{hx+ky}$, where $k = \dfrac{(h^2 + h - 1)}{4h}$

$$P.I. = \frac{1}{F(D, D')}e^{ax+by} = \frac{1}{(D^2 - 4DD' + D - 1)}e^{3x-2y} = \frac{1}{(3^2 - 4.3.(-2) + 3 - 1)}e^{3x-2y} = \frac{1}{35}e^{3x-2y}.$$

Hence the required solution is

$$z = \sum Ae^{hx+ky} + \frac{1}{35}e^{3x-2y},\text{where } k = \frac{(h^2 + h - 1)}{4h}.$$

$$\implies k = \frac{(h^2 + h - 1)}{4h}.$$

Thus $C.F. = \sum Ae^{hx+ky}$, where $k = \dfrac{(h^2 + h - 1)}{4h}$

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{(D^2 - 4DD' + D - 1)} e^{3x-2y} = \frac{1}{(3^2 - 4.3.(-2) + 3 - 1)} e^{3x-2y} = \frac{1}{35} e^{3x-2y}.$$

Hence the required solution is

$$z = \sum Ae^{hx+ky} + \frac{1}{35} e^{3x-2y}, \text{where } k = \frac{(h^2 + h - 1)}{4h}.$$

$$\implies k = \frac{(h^2 + h - 1)}{4h}.$$

Thus $C.F. = \sum Ae^{hx+ky}$, where $k = \frac{(h^2 + h - 1)}{4h}$

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{(D^2 - 4DD' + D - 1)} e^{3x-2y} = \frac{1}{(3^2 - 4.3.(-2) + 3 - 1)} e^{3x-2y} = \frac{1}{35} e^{3x-2y}.$$

Hence the required solution is

$$z = \sum Ae^{hx+ky} + \frac{1}{35} e^{3x-2y}, \text{where } k = \frac{(h^2 + h - 1)}{4h}.$$

$$\implies k = \frac{(h^2+h-1)}{4h}.$$

Thus $C.F. = \sum Ae^{hx+ky}$, where $k = \dfrac{(h^2+h-1)}{4h}$

$$P.I. = \frac{1}{F(D,D')}e^{ax+by} = \frac{1}{(D^2-4DD'+D-1)}e^{3x-2y} = \frac{1}{(3^2-4.3.(-2)+3-1)}e^{3x-2y} = \frac{1}{35}e^{3x-2y}.$$

Hence the required solution is

$$z = \sum Ae^{hx+ky} + \frac{1}{35}e^{3x-2y}, \text{where } k = \frac{(h^2+h-1)}{4h}.$$

$$\implies k = \frac{(h^2 + h - 1)}{4h}.$$

Thus $C.F. = \sum Ae^{hx+ky}$, where $k = \dfrac{(h^2 + h - 1)}{4h}$

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{(D^2 - 4DD' + D - 1)} e^{3x-2y} = \frac{1}{(3^2 - 4.3.(-2) + 3 - 1)} e^{3x-2y} = \frac{1}{35} e^{3x-2y}.$$

Hence the required solution is

$$z = \sum Ae^{hx+ky} + \frac{1}{35} e^{3x-2y}, \text{where } k = \frac{(h^2 + h - 1)}{4h}.$$

$$\implies k = \frac{(h^2 + h - 1)}{4h}.$$

Thus $C.F. = \sum Ae^{hx+ky}$, where $k = \dfrac{(h^2 + h - 1)}{4h}$

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{(D^2 - 4DD' + D - 1)} e^{3x-2y} = \frac{1}{(3^2 - 4.3.(-2) + 3 - 1)} e^{3x-2y} = \frac{1}{35} e^{3x-2y}.$$

Hence the required solution is

$$z = \sum Ae^{hx+ky} + \frac{1}{35} e^{3x-2y}, \text{where } k = \frac{(h^2 + h - 1)}{4h}.$$

$$\implies k = \frac{(h^2 + h - 1)}{4h}.$$

Thus $C.F. = \sum Ae^{hx+ky}$, where $k = \dfrac{(h^2 + h - 1)}{4h}$

$$P.I. = \frac{1}{F(D, D')}e^{ax+by} = \frac{1}{(D^2 - 4DD' + D - 1)}e^{3x-2y} = \frac{1}{(3^2 - 4.3.(-2) + 3 - 1)}e^{3x-2y} = \frac{1}{35}e^{3x-2y}.$$

Hence the required solution is

$$z = \sum Ae^{hx+ky} + \frac{1}{35}e^{3x-2y}, \text{where } k = \frac{(h^2 + h - 1)}{4h}.$$

$$\implies k = \frac{(h^2 + h - 1)}{4h}.$$

Thus $C.F. = \sum A e^{hx+ky}$, where $k = \dfrac{(h^2 + h - 1)}{4h}$

$$P.I. = \frac{1}{F(D, D')} e^{ax+by} = \frac{1}{(D^2 - 4DD' + D - 1)} e^{3x-2y} = \frac{1}{(3^2 - 4.3.(-2) + 3 - 1)} e^{3x-2y} = \frac{1}{35} e^{3x-2y}.$$

Hence the required solution is

$$z = \sum A e^{hx+ky} + \frac{1}{35} e^{3x-2y}, \text{where } k = \frac{(h^2 + h - 1)}{4h}.$$

Exercise

Solve the following PDE:

(1) $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$ **Ans.** $z = e^{x} f_1(y+x) + e^{2x} f_2(y+x) + (1/2)e^{2x-1}$.

(2) $(D^3 - 3DD' + D + 1)z = e^{2x+3y}$ **Ans.** $z = \sum Ae^{hx+ky} - \frac{1}{7}e^{2x+3y}$, where $k = \frac{(h^3+h+1)}{3h}$.

(3) $(D^2 - D'^2 - 3D')z = e^{x+2y}$ **Ans.** $z = \sum Ae^{hx+ky} - \frac{1}{9}e^{x+2y}$, where $h = \sqrt{k^2+3k}$.

(4) $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y}$ **Ans.** $z = e^{-2x} f_1(y+x) + e^{x} f_2(y-x) - (1/4)e^{x-y}$.

Exercise

Solve the following PDE:

(1) $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$ **Ans.** $z = e^x f_1(y + x) + e^{2x} f_2(y + x) + (1/2)e^{2x-1}$.

(2) $(D^3 - 3DD' + D + 1)z = e^{2x+3y}$ **Ans.** $z = \sum Ae^{hx+ky} - \frac{1}{7}e^{2x+3y}$, where $k = \frac{(h^3 + h + 1)}{3h}$.

(3) $(D^2 - D'^2 - 3D')z = e^{x+2y}$ **Ans.** $z = \sum Ae^{hx+ky} - \frac{1}{9}e^{x+2y}$, where $h = \sqrt{k^2 + 3k}$.

(4) $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y}$ **Ans.** $z = e^{-2x} f_1(y + x) + e^x f_2(y - x) - (1/4)e^{x-y}$.

Exercise

Solve the following PDE:

(1) $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$ **Ans.** $z = e^{x} f_1(y + x) + e^{2x} f_2(y + x) + (1/2)e^{2x-1}$.

(2) $(D^3 - 3DD' + D + 1)z = e^{2x+3y}$ **Ans.** $z = \sum Ae^{hx+ky} - \dfrac{1}{7}e^{2x+3y}$, where $k = \dfrac{(h^3 + h + 1)}{3h}$.

(3) $(D^2 - D'^2 - 3D')z = e^{x+2y}$ **Ans.** $z = \sum Ae^{hx+ky} - \dfrac{1}{9}e^{x+2y}$, where $h = \sqrt{k^2 + 3k}$.

(4) $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y}$ **Ans.** $z = e^{-2x} f_1(y + x) + e^{x} f_2(y - x) - (1/4)e^{x-y}$.

Exercise

Solve the following PDE:

(1) $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$ **Ans.** $z = e^{x} f_1(y+x) + e^{2x} f_2(y+x) + (1/2)e^{2x-1}$.

(2) $(D^3 - 3DD' + D + 1)z = e^{2x+3y}$ **Ans.** $z = \sum Ae^{hx+ky} - \frac{1}{7}e^{2x+3y}$, where $k = \frac{(h^3+h+1)}{3h}$.

(3) $(D^2 - D'^2 - 3D')z = e^{x+2y}$ **Ans.** $z = \sum Ae^{hx+ky} - \frac{1}{9}e^{x+2y}$, where $h = \sqrt{k^2+3k}$.

(4) $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y}$ **Ans.** $z = e^{-2x} f_1(y+x) + e^{x} f_2(y-x) - (1/4)e^{x-y}$.

Case-II: When $\phi(x, y) = \sin(ax + by)$ or $\cos(ax + by)$.

Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D, D')} \sin(ax + by) \text{ or } \frac{1}{F(D, D')} \cos(ax + by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.

Solution: The given equation can be written as linear factors$(D + 1)(D + D' - 1)z = \sin(x + 2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x} f_1(y) + e^{x} f_2(y - x).$$

Case-II: When $\phi(x, y) = \sin(ax + by)$ or $\cos(ax + by)$.
Then, we get the P.I., by replacing D^2 by $-a^2$, D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D, D')} \sin(ax + by) \text{ or } \frac{1}{F(D, D')} \cos(ax + by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.

Solution: The given equation can be written as linear factors$(D + 1)(D + D' - 1)z = \sin(x + 2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x} f_1(y) + e^x f_2(y - x).$$

Case-II: When $\phi(x,y) = \sin(ax+by)$ or $\cos(ax+by)$.
Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D,D')}\sin(ax+by) \text{ or } \frac{1}{F(D,D')}\cos(ax+by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x+2y)$.

Solution: The given equation can be written as linear factors$(D+1)(D+D'-1)z = \sin(x+2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x}f_1(y) + e^{x}f_2(y-x).$$

Case-II: When $\phi(x,y) = \sin(ax+by)$ or $\cos(ax+by)$.
Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D,D')}\sin(ax+by) \text{ or } \frac{1}{F(D,D')}\cos(ax+by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x+2y)$.

Solution: The given equation can be written as linear factors$(D+1)(D+D'-1)z = \sin(x+2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x}f_1(y) + e^{x}f_2(y-x).$$

Case-II: When $\phi(x,y) = \sin(ax+by)$ or $\cos(ax+by)$.
Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D,D')}\sin(ax+by) \text{ or } \frac{1}{F(D,D')}\cos(ax+by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x+2y)$.

Solution: The given equation can be written as linear factors$(D+1)(D+D'-1)z = \sin(x+2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x}f_1(y) + e^{x}f_2(y-x).$$

Case-II: When $\phi(x, y) = \sin(ax + by)$ or $\cos(ax + by)$. Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D, D')} \sin(ax + by) \text{ or } \frac{1}{F(D, D')} \cos(ax + by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.

Solution: The given equation can be written as linear factors$(D + 1)(D + D' - 1)z = \sin(x + 2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x} f_1(y) + e^{x} f_2(y - x).$$

Case-II: When $\phi(x,y) = \sin(ax+by)$ or $\cos(ax+by)$.
Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D,D')}\sin(ax+by) \text{ or } \frac{1}{F(D,D')}\cos(ax+by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x+2y)$.

Solution: The given equation can be written as linear factors$(D+1)(D+D'-1)z = \sin(x+2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x}f_1(y) + e^{x}f_2(y-x).$$

Case-II: When $\phi(x,y) = \sin(ax+by)$ or $\cos(ax+by)$.
Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D,D')}\sin(ax+by) \text{ or } \frac{1}{F(D,D')}\cos(ax+by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x+2y)$.

Solution: The given equation can be written as linear factors$(D+1)(D+D'-1)z = \sin(x+2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x}f_1(y) + e^{x}f_2(y-x).$$

Case-II: When $\phi(x, y) = \sin(ax + by)$ or $\cos(ax + by)$.
Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D, D')}\sin(ax + by) \text{ or } \frac{1}{F(D, D')}\cos(ax + by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.

Solution: The given equation can be written as linear factors$(D + 1)(D + D' - 1)z = \sin(x + 2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x} f_1(y) + e^{x} f_2(y - x).$$

Case-II: When $\phi(x,y) = \sin(ax+by)$ or $\cos(ax+by)$.
Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D,D')}\sin(ax+by) \text{ or } \frac{1}{F(D,D')}\cos(ax+by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x+2y)$.

Solution: The given equation can be written as linear factors$(D+1)(D+D'-1)z = \sin(x+2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x} f_1(y) + e^{x} f_2(y-x).$$

Case-II: When $\phi(x,y) = \sin(ax+by)$ or $\cos(ax+by)$.
Then, we get the P.I., by replacing D^2 by $-a^2$,D'^2 by $-b^2$ and DD' by $-ab$ in

$$P.I. = \frac{1}{F(D,D')}\sin(ax+by) \text{ or } \frac{1}{F(D,D')}\cos(ax+by),$$

provided denominator should not be zero.

Example

Solve the PDE $(D^2 + DD' + D' - 1)z = \sin(x+2y)$.

Solution: The given equation can be written as linear factors$(D+1)(D+D'-1)z = \sin(x+2y)$. Hence it's complementary function (C.F.) is

$$C.F. = e^{-x}f_1(y) + e^x f_2(y-x).$$

Now

$$P.I. = \frac{1}{F(D,D')}\sin(x+2y) = \frac{1}{(D^2+DD'+D'-1)}\sin(x+2y) = \frac{1}{(-1^2+(-1.2)+D'-1)}\sin(x+2y) = \frac{1}{D'-4}\sin(x+2y).$$

$$P.I. = (D'+4)\frac{1}{D'^2-4^2}\sin(x+2y) \implies (D'+4)\frac{1}{-2^2-16}\sin(x+2y).$$

Now

$$P.I. = \frac{1}{F(D,D')}\sin(x+2y) = \frac{1}{(D^2+DD'+D'-1)}\sin(x+2y) = \frac{1}{(-1^2+(-1.2)+D'-1)}\sin(x+2y) = \frac{1}{D'-4}\sin(x+2y).$$

$$P.I. = (D'+4)\frac{1}{D'^2-4^2}\sin(x+2y) \implies (D'+4)\frac{1}{-2^2-16}\sin(x+2y).$$

Now

$$P.I. = \frac{1}{F(D,D')}\sin(x+2y) = \frac{1}{(D^2+DD'+D'-1)}\sin(x+2y) = \frac{1}{(-1^2+(-1.2)+D'-1)}\sin(x+2y) = \frac{1}{D'-4}\sin(x+2y).$$

$$P.I. = (D'+4)\frac{1}{D'^2-4^2}\sin(x+2y) \implies (D'+4)\frac{1}{-2^2-16}\sin(x+2y).$$

Now

$$P.I. = \frac{1}{F(D, D')}\sin(x+2y) = \frac{1}{(D^2 + DD' + D' - 1)}\sin(x+2y) = \frac{1}{(-1^2 + (-1.2) + D' - 1)}\sin(x+2y) = \frac{1}{D' - 4}\sin(x+2y).$$

$$P.I. = (D'+4)\frac{1}{D'^2 - 4^2}\sin(x+2y) \implies (D'+4)\frac{1}{-2^2 - 16}\sin(x+2y).$$

Now

$$P.I. = \frac{1}{F(D,D')}\sin(x+2y) = \frac{1}{(D^2+DD'+D'-1)}\sin(x+2y) = \frac{1}{(-1^2+(-1.2)+D'-1)}\sin(x+2y) = \frac{1}{D'-4}\sin(x+2y).$$

$$P.I. = (D'+4)\frac{1}{D'^2-4^2}\sin(x+2y) \implies (D'+4)\frac{1}{-2^2-16}\sin(x+2y).$$

$$P.I. = -\frac{1}{20}(D' + 4)\sin(x + 2y) \implies -\frac{1}{20}\left[D'\sin(x + 2y) + 4\sin(x + 2y)\right].$$

$$P.I. = -\frac{1}{20}\left[2\cos(x + 2y) + 4\sin(x + 2y)\right].$$

Hence the required solution is

$$z = e^{-x}f_1(y) + e^{x}f_2(y - x) - \frac{1}{10}\left[\cos(x + 2y) + 2\sin(x + 2y)\right].$$

$$P.I. = -\frac{1}{20}(D'+4)\sin(x+2y) \implies -\frac{1}{20}\left[D'\sin(x+2y)+4\sin(x+2y)\right].$$

$$P.I. = -\frac{1}{20}\left[2\cos(x+2y)+4\sin(x+2y)\right].$$

Hence the required solution is

$$z = e^{-x}f_1(y)+e^{x}f_2(y-x)-\frac{1}{10}\left[\cos(x+2y)+2\sin(x+2y)\right].$$

$$P.I. = -\frac{1}{20}(D' + 4)\sin(x + 2y) \implies$$
$$-\frac{1}{20}\left[D'\sin(x + 2y) + 4\sin(x + 2y)\right].$$

$$P.I. = -\frac{1}{20}\left[2\cos(x + 2y) + 4\sin(x + 2y)\right].$$

Hence the required solution is

$$z = e^{-x}f_1(y) + e^{x}f_2(y - x) - \frac{1}{10}\left[\cos(x + 2y) + 2\sin(x + 2y)\right].$$

$$P.I. = -\frac{1}{20}(D'+4)\sin(x+2y) \implies -\frac{1}{20}\left[D'\sin(x+2y)+4\sin(x+2y)\right].$$

$$P.I. = -\frac{1}{20}\left[2\cos(x+2y)+4\sin(x+2y)\right].$$

Hence the required solution is

$$z = e^{-x}f_1(y)+e^{x}f_2(y-x)-\frac{1}{10}\left[\cos(x+2y)+2\sin(x+2y)\right].$$

$$P.I. = -\frac{1}{20}(D'+4)\sin(x+2y) \implies -\frac{1}{20}\left[D'\sin(x+2y)+4\sin(x+2y)\right].$$

$$P.I. = -\frac{1}{20}\left[2\cos(x+2y)+4\sin(x+2y)\right].$$

Hence the required solution is

$$z = e^{-x}f_1(y)+e^{x}f_2(y-x) - \frac{1}{10}\left[\cos(x+2y)+2\sin(x+2y)\right].$$

$$P.I. = -\frac{1}{20}(D' + 4)\sin(x + 2y) \implies -\frac{1}{20}\left[D'\sin(x+2y) + 4\sin(x+2y)\right].$$

$$P.I. = -\frac{1}{20}\left[2\cos(x+2y) + 4\sin(x+2y)\right].$$

Hence the required solution is

$$z = e^{-x}f_1(y) + e^{x}f_2(y-x) - \frac{1}{10}\left[\cos(x+2y) + 2\sin(x+2y)\right].$$

Example

Solve the PDE $(D - D'^2)z = \cos(x - 3y)$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

So that $Dz = \sum Ahe^{hx+ky}$and $D'^2z = \sum Ak^2e^{hx+ky}$.By Putting these values in given equation $(D - D'^2)z = 0$, we have

$$\sum Ahe^{hx+ky} - \sum Ak^2e^{hx+ky} = 0 \implies \sum A(h-k^2)e^{hx+ky} = 0.$$

Example

Solve the PDE $(D - D'^2)z = \cos(x - 3y)$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

So that $Dz = \sum Ahe^{hx+ky}$ and $D'^2z = \sum Ak^2e^{hx+ky}$.By Putting these values in given equation $(D - D'^2)z = 0$, we have

$$\sum Ahe^{hx+ky} - \sum Ak^2e^{hx+ky} = 0 \implies \sum A(h-k^2)e^{hx+ky} = 0.$$

Example

Solve the PDE $(D - D'^2)z = \cos(x - 3y)$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

So that $Dz = \sum Ahe^{hx+ky}$and $D'^2z = \sum Ak^2e^{hx+ky}$.By Putting these values in given equation $(D - D'^2)z = 0$, we have

$$\sum Ahe^{hx+ky} - \sum Ak^2e^{hx+ky} = 0 \implies \sum A(h-k^2)e^{hx+ky} = 0.$$

Example

Solve the PDE $(D - D'^2)z = \cos(x - 3y)$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

So that $Dz = \sum Ahe^{hx+ky}$ and $D'^2 z = \sum Ak^2 e^{hx+ky}$.By Putting these values in given equation $(D - D'^2)z = 0$, we have

$$\sum Ahe^{hx+ky} - \sum Ak^2 e^{hx+ky} = 0 \implies \sum A(h - k^2)e^{hx+ky} = 0.$$

Example

Solve the PDE $(D - D'^2)z = \cos(x - 3y)$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

So that $Dz = \sum Ahe^{hx+ky}$and $D'^2 z = \sum Ak^2 e^{hx+ky}$.By Putting these values in given equation $(D - D'^2)z = 0$, we have

$$\sum Ahe^{hx+ky} - \sum Ak^2 e^{hx+ky} = 0 \implies \sum A(h-k^2)e^{hx+ky} = 0.$$

Example

Solve the PDE $(D - D'^2)z = \cos(x - 3y)$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

So that $Dz = \sum Ahe^{hx+ky}$and $D'^2z = \sum Ak^2e^{hx+ky}$.By Putting these values in given equation $(D - D'^2)z = 0$, we have

$$\sum Ahe^{hx+ky} - \sum Ak^2e^{hx+ky} = 0 \implies \sum A(h-k^2)e^{hx+ky} = 0.$$

Example

Solve the PDE $(D - D'^2)z = \cos(x - 3y)$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

So that $Dz = \sum Ahe^{hx+ky}$and $D'^2 z = \sum Ak^2 e^{hx+ky}$.By Putting these values in given equation $(D - D'^2)z = 0$, we have

$$\sum Ahe^{hx+ky} - \sum Ak^2 e^{hx+ky} = 0 \implies \sum A(h - k^2)e^{hx+ky} = 0.$$

Example

Solve the PDE $(D - D'^2)z = \cos(x - 3y)$.

Solution: The given equation can not be written as linear factors. Hence it's complementary function (C.F.) is taken as a trial solution

$$z = \sum Ae^{hx+ky}.$$

So that $Dz = \sum Ahe^{hx+ky}$and $D'^2 z = \sum Ak^2 e^{hx+ky}$.By Putting these values in given equation $(D - D'^2)z = 0$, we have

$$\sum Ahe^{hx+ky} - \sum Ak^2 e^{hx+ky} = 0 \implies \sum A(h-k^2)e^{hx+ky} = 0.$$

$(D - D'^2)z = 0$, we have

$$h - k^2 = 0 \implies h = k^2.$$

Hence

$$C.F. = \sum A e^{k^2 x + ky}.$$

$$P.I. = \frac{1}{F(D, D')} \cos(ax + by) = \frac{1}{(D - D'^2)} \cos(x - 3y) = \frac{1}{(D - (-3^2))} \cos(x - 3y) = \frac{1}{D + 9} \cos(x - 3y).$$

$$P.I. = (D - 9) \frac{1}{D^2 - 9^2} \cos(x - 3y) \implies (D - 9) \frac{1}{-1^2 - 81} \cos(x - 3y).$$

$(D - D'^2)z = 0$, we have

$$h - k^2 = 0 \implies h = k^2.$$

Hence

$$C.F. = \sum Ae^{k^2x+ky}.$$

$$P.I. = \frac{1}{F(D,D')}\cos(ax+by) = \frac{1}{(D-D'^2)}\cos(x-3y) = \frac{1}{(D-(-3^2))}\cos(x-3y) = \frac{1}{D+9}\cos(x-3y).$$

$$P.I. = (D-9)\frac{1}{D^2-9^2}\cos(x-3y) \implies (D-9)\frac{1}{-1^2-81}\cos(x-3y).$$

$(D - D'^2)z = 0$, we have

$$h - k^2 = 0 \implies h = k^2.$$

Hence

$$C.F. = \sum Ae^{k^2x+ky}.$$

$$P.I. = \frac{1}{F(D,D')}\cos(ax+by) = \frac{1}{(D-D'^2)}\cos(x-3y) = \frac{1}{(D-(-3^2))}\cos(x-3y) = \frac{1}{D+9}\cos(x-3y).$$

$$P.I. = (D-9)\frac{1}{D^2-9^2}\cos(x-3y) \implies (D-9)\frac{1}{-1^2-81}\cos(x-3y).$$

$(D - D'^2)z = 0$, we have

$$h - k^2 = 0 \implies h = k^2.$$

Hence

$$C.F. = \sum Ae^{k^2x+ky}.$$

$$P.I. = \frac{1}{F(D,D')}\cos(ax+by) = \frac{1}{(D-D'^2)}\cos(x-3y) = \frac{1}{(D-(-3^2))}\cos(x-3y) = \frac{1}{D+9}\cos(x-3y).$$

$$P.I. = (D-9)\frac{1}{D^2-9^2}\cos(x-3y) \implies (D-9)\frac{1}{-1^2-81}\cos(x-3y).$$

$(D - D'^2)z = 0$, we have

$$h - k^2 = 0 \implies h = k^2.$$

Hence

$$C.F. = \sum Ae^{k^2x+ky}.$$

$$P.I. = \frac{1}{F(D, D')} \cos(ax + by) = \frac{1}{(D - D'^2)} \cos(x - 3y) = \frac{1}{(D - (-3^2))} \cos(x - 3y) = \frac{1}{D + 9} \cos(x - 3y).$$

$$P.I. = (D - 9)\frac{1}{D^2 - 9^2} \cos(x - 3y) \implies (D - 9)\frac{1}{-1^2 - 81} \cos(x - 3y).$$

Homogeneous Linear Partial Differential ...

Dr. G.K. Prajapati

LNJPIT, Chapra

Introduction

$(D - D'^2)z = 0$, we have

$$h - k^2 = 0 \implies h = k^2.$$

Hence

$$C.F. = \sum Ae^{k^2x+ky}.$$

$$P.I. = \frac{1}{F(D,D')}\cos(ax+by) = \frac{1}{(D-D'^2)}\cos(x-3y) = \frac{1}{(D-(-3^2))}\cos(x-3y) = \frac{1}{D+9}\cos(x-3y).$$

$$P.I. = (D-9)\frac{1}{D^2-9^2}\cos(x-3y) \implies (D-9)\frac{1}{-1^2-81}\cos(x-3y).$$

$(D - D'^2)z = 0$, we have

$$h - k^2 = 0 \implies h = k^2.$$

Hence

$$C.F. = \sum Ae^{k^2x+ky}.$$

$$P.I. = \frac{1}{F(D, D')}\cos(ax + by) = \frac{1}{(D - D'^2)}\cos(x - 3y) = \frac{1}{(D - (-3^2))}\cos(x - 3y) = \frac{1}{D + 9}\cos(x - 3y).$$

$$P.I. = (D - 9)\frac{1}{D^2 - 9^2}\cos(x - 3y) \implies (D - 9)\frac{1}{-1^2 - 81}\cos(x - 3y).$$

$(D - D'^2)z = 0$, we have

$$h - k^2 = 0 \implies h = k^2.$$

Hence

$$C.F. = \sum Ae^{k^2x+ky}.$$

$$P.I. = \frac{1}{F(D, D')}\cos(ax+by) = \frac{1}{(D - D'^2)}\cos(x-3y) = \frac{1}{(D-(-3^2))}\cos(x-3y) = \frac{1}{D+9}\cos(x-3y).$$

$$P.I. = (D-9)\frac{1}{D^2-9^2}\cos(x-3y) \implies (D-9)\frac{1}{-1^2-81}\cos(x-3y).$$

$(D-D'^2)z=0$, we have

$$h-k^2=0 \implies h=k^2.$$

Hence

$$C.F.=\sum Ae^{k^2x+ky}.$$

$$P.I.=\frac{1}{F(D,D')}\cos(ax+by)=\frac{1}{(D-D'^2)}\cos(x-3y)=\frac{1}{(D-(-3^2))}\cos(x-3y)=\frac{1}{D+9}\cos(x-3y).$$

$$P.I.=(D-9)\frac{1}{D^2-9^2}\cos(x-3y) \implies (D-9)\frac{1}{-1^2-81}\cos(x-3y).$$

$$P.I. = -\frac{1}{82}(D-9)\cos(x-3y) \implies -\frac{1}{82}\left[D\cos(x-3y) - 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}\left[-\sin(x-3y) - 9\cos(x-3y)\right] \implies = \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

Hence the required solution is

$$z = \sum Ae^{k^2x+ky} + \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}(D-9)\cos(x-3y) \implies -\frac{1}{82}\left[D\cos(x-3y) - 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}\left[-\sin(x-3y) - 9\cos(x-3y)\right] \implies = \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

Hence the required solution is

$$z = \sum Ae^{k^2x+ky} + \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}(D-9)\cos(x-3y) \implies -\frac{1}{82}\left[D\cos(x-3y) - 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}\left[-\sin(x-3y) - 9\cos(x-3y)\right] \implies = \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

Hence the required solution is

$$z = \sum Ae^{k^2x+ky} + \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}(D-9)\cos(x-3y) \implies -\frac{1}{82}\left[D\cos(x-3y) - 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}\left[-\sin(x-3y) - 9\cos(x-3y)\right] \implies = \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

Hence the required solution is

$$z = \sum Ae^{k^2x+ky} + \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}(D-9)\cos(x-3y) \implies -\frac{1}{82}\left[D\cos(x-3y) - 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}\left[-\sin(x-3y) - 9\cos(x-3y)\right] \implies = \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

Hence the required solution is

$$z = \sum Ae^{k^2x+ky} + \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}(D-9)\cos(x-3y) \implies -\frac{1}{82}\left[D\cos(x-3y) - 9\cos(x-3y)\right].$$

$$P.I. = -\frac{1}{82}\left[-\sin(x-3y) - 9\cos(x-3y)\right] \implies = \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

Hence the required solution is

$$z = \sum Ae^{k^2x+ky} + \frac{1}{82}\left[\sin(x-3y) + 9\cos(x-3y)\right].$$

Exercise

Solve the following PDE:

(1) $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial z}{\partial y} - z = \cos(x+2y)$ **Ans.** $z = e^x f_1(y) + e^{-x} f_2(y+x) + (1/2)\sin(x+2y)$.

(2) $(D^2 - DD' - 2D)z = \sin(3x+4y)$ **Ans.** $z = f_1(y) + e^{2x} f_2(y+x) + (1/15)\left[\sin(3x+4y) + 2\cos(3x+4y)\right]$.

(3) $(D - D' - 1)(D - D' - 2)z = \sin(2x+3y)$ **Ans.** $z = e^x f_1(y+x) + e^{2x} f_2(y+x) + (1/10)\left[\sin(2x+3y) - 12\cos(2x+3y)\right]$.

(4) $(D^2 - D')z = \cos(3x-y)$ **Ans.** $z = \sum A e^{hx+h^2 y} - \dfrac{1}{82}\left[-\sin(3x-y) + 9\cos(3x-y)\right]$.

Thanks !!!