

Lok Nayak Jai Prakash Institute of Technology Chapra, Bihar-841302

Homogeneous Linear Partial Differential ...

> Dr. G.K. Prajapati

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Introduction

Mathematics-II (Differential Equations) Lecture Notes April 10, 2020

by

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Working rule for finding C.F. of irreducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given irreducible non-homogeneous linear partial differential equations with constants coefficients be $F(D, D')z = \phi(x, y)$ **Step-I**: If necessary Factorize F(D, D') in the form $F_1(D, D')F_2(D, D')$, where $F_1(D, D')$ consists of product of linear factors in D, D' and $F_2(D, D')$ consists of product of irreducible factors in D, D'.

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$\mbox{Step-II}:$ Write the part of C.F. of linear factors $F_1(D,D')$ as usual method

Step-III: Write the part of C.F. of irreducible factors $F_2(D, D')$ by taking a trial solution

$$C.F. = \sum Ae^{hx+ky},$$

where A, h and k are arbitrary constants such that F(h, k) = 0**Step-IV**: Adding the part of C.F. of reducible factors $F_1(D, D')$, obtained in Step-II and part of C.F. of irreducible factors $F_2(D, D')$, obtained in Step-III.

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Solve the PDE $(D - D'^2)z = 0.$

Solution: Here $D - D'^2$ is not a linear factors in D and D'. Let the trial solution of given equation is

$$z = \sum A e^{hx + ky}$$

Then $Dz = Ahe^{hx+ky}$ and $D'^2z = Ak^2e^{hx+ky}$. Putting these values in the given equation, we get

$$Ahe^{hx+ky} - Ak^2e^{hx+ky} = 0 \implies A(h-k^2)e^{hx+ky} = 0$$

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Replacing h by $k^2, \, {\rm the \; most \; general \; solution \; of the given equation is }$

$$z = \sum A e^{k^2 x + ky},$$

where A and k are arbitrary constant.

Image: A matched block

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$$(D - 2D' - 1)(D - 2D'^2 - 1)z = 0.$$

Solution: Here (D - 2D' - 1) is a linear factors in D and D'. Therefore its complementary function (C.F.) is

$$e^x f_1(y+2x),$$

where f_1 is an arbitrary function. To find the complementary function (C.F.) corresponding factor $(D - 2D'^2 - 1)z$. Let the trial solution of this factor is

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Replacing h by $2k^2 + 1$, the complementary function (C.F.) corresponding factor $(D - 2D'^2 - 1)z$ is $C.F. = \sum Ae^{(k^2+1)x+ky}$. Now the required general solution of the given equation is

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Solve the PDE
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$$h^2 - k = 0 \implies k = h^2.$$

Replacing k by h^2 , the complementary function (C.F.) corresponding factor $(D^2 - D')z$ is $C.F. = \sum Ae^{hx+h^2y}$.

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$$z = \sum A e^{hx + h^2 y} + \sum A_1 e^{h_1 x + 2h_1^2 y},$$

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Thanks !!!

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