Mathematics-II (Differential Equations)
Lecture Notes
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by

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Introduction

Working rule for finding C.F. of irreducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given irreducible non-homogeneous linear partial differential equations with constants coefficients be $F(D, D')z = \phi(x, y)$

Step-I: If necessary Factorize $F(D, D')$ in the form $F_1(D, D')F_2(D, D')$, where $F_1(D, D')$ consists of product of linear factors in D, D' and $F_2(D, D')$ consists of product of irreducible factors in D, D'.

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Step-II: Write the part of C.F. of linear factors $F_1(D, D')$ as usual method

Step-III: Write the part of C.F. of irreducible factors $F_2(D, D')$ by taking a trial solution

$$C.F. = \sum Ae^{hx+ky},$$

where A, h and k are arbitrary constants such that $F(h, k) = 0$

Step-IV: Adding the part of C.F. of reducible factors $F_1(D, D')$, obtained in Step-II and part of C.F. of irreducible factors $F_2(D, D')$, obtained in Step-III.

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Example

Solve the PDE $(D - D'^2)z = 0$.

Solution: Here $D - D'^2$ is not a linear factors in D and D'. Let the trial solution of given equation is

$$z = \sum Ae^{hx+ky}$$

Then $Dz = Ahe^{hx+ky}$ and $D'^2z = Ak^2e^{hx+ky}$.Putting these values in the given equation, we get

$$Ahe^{hx+ky} - Ak^2e^{hx+ky} = 0 \implies A(h - k^2)e^{hx+ky} = 0$$

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Solve the PDE $(D-2D'-1)(D-2D'^2-1)z=0.$

Solution: Here $(D-2D'-1)$ is a linear factors in D and D'.Therefore its complementary function (C.F.) is

$$e^x f_1(y+2x),$$

where f_1 is an arbitrary function.To find the complementary function (C.F.) corresponding factor $(D-2D'^2-1)z$.Let the trial solution of this factor is

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$$h - 2k^2 - 1 = 0 \implies h = 2k^2 + 1.$$

Replacing h by $2k^2 + 1$, the complementary function (C.F.) corresponding factor $(D - 2D'^2 - 1)z$ is$C.F. = \sum Ae^{(k^2+1)x+ky}$. Now the required general solution of the given equation is

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Example

Solve the PDE $(2D^4 - 3D^2D' + D'^2)z = 0$.

Solution: Given equation can be written as $(2D^2 - D')(D^2 - D')z = 0$.To find the complementary function (C.F.) corresponding factor $(D^2 - D')z$.Let the trial solution of this factor is

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$$A_1 (2h_1^2 - k_1) e^{h_1 x + k_1 y} = 0$$

Again to find the complementary function (C.F.) corresponding factor $(2D^2 - D')z$.Let the trial solution of this factor is

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Replacing k_1 by $2h_1^2$, the complementary function (C.F.) corresponding factor $(2D^2 - D')z$ is$C.F. = \sum A_1 e^{h_1 x + 2h_1^2 y}$.
Now the required general solution of the given equation is

$$z = \sum A e^{hx + h^2 y} + \sum A_1 e^{h_1 x + 2h_1^2 y},$$

where A, A_1, h and h_1 are arbitrary constant.

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Thanks !!!