

Lok Nayak Jai Prakash Institute of Technology Chapra, Bihar-841302

Non-[Homogeneous](#page-12-0) Linear Partial Differential ...

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Mathematics-II (Differential Equations) Lecture Notes April 8, 2020

by

Dr. G.K.Prajapati Department of Applied Science and Humanities LNJPIT, Chapra, Bihar-841302

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Non-Homogeneous Linear Partial Differential Equations with Constants Coefficients

Definition

A linear partial differential equations with constant coefficients, which are not homogeneous are called Non-homogeneous.

$$
2\frac{\partial^3 z}{\partial z^3} - 3\frac{\partial^2 z}{\partial z^2} + \frac{\partial z}{\partial z} + 2z = x + 2y
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$$
\frac{\partial^3 z}{\partial z^3} + \frac{\partial z}{\partial z} - 4z = \sin(x + 2y)
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Definition

A linear differential operator $F(D, D')$ is known as reducible, if it can be written as the product of linear factors of the form $\bm{a}D+\bm{b}D'+\bm{c},$ where $a,$ b and c are constants. $F(D,D')$ is known as irreducible, if it is not reducible.

 $D^2-D^{\prime 2}$ is reducible because it can be written as a linear factor $(D^2 - D'^2) = (D - D')(D + D')$

 $D^3D'-DD'^3$ is reducible because it can be written as a linear factor $D^3D^\prime - DD^{\prime 3} = DD^\prime(D-D^\prime)(D+D^\prime)$

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Working rule for finding C.F. of reducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given reducible non-homogeneous linear partial differential equations with constants coefficients be $F(D, D')z = \phi(x, y)$ **Step-I**: Factorize $F(D, D')$ into linear factors. Step-II: Corresponding to each non-repeated factor $(bD - aD' - c)$, the part of complementary function is taken as $e^{(cx/b)}f_1(by+ax)$, if $b\neq 0$. **Step-III**: Corresponding to repeated factor $(bD - aD' - c)^r$, the part of complementary function is taken as $e^{(cx/b)} [f_1(by + ax) + xf_2(by + ax) + x^2f_3(by + ax) + ...$ $+x^{(r-1)}f_r(by+ax)$, if $b \neq 0$. $4.11 \times 1.00 \times 1.7 \times 1.4$

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Step-IV: Corresponding to each non-repeated factor $(bD - aD' - c)$, the part of complementary function is taken as $e^{-(cy/a)}f -_1 (by + ax)$, if $a \neq 0$.

Step-V: Corresponding to repeated factor $(bD - aD' - c)^{r}$, the part of complementary function is taken as $e^{-(cy/a)} [f_1(by + ax) + yf_2(by + ax) + y^2f_3(by + ax) + ...$ $+y^{(r-1)}f_r(by+ax)\big]$, if $a\neq 0, \, f_1, f_2, f_3,...,f_r$ are arbitrary functions.

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Example

Solve the PDE $(D^2 - D'^2 + D - D')z = 0.$

Solution: The given PDE $(D^2 - D'^2 + D - D')z = 0$ is reducible because it can be written as a linear factor

$$
[(D - D')(D + D') + D - D']z = 0 \implies
$$

$$
(D - D')(D + D' + 1)z = 0.
$$

By comparing $(D - D')$ with $(bD - aD' - c)$, we get $b = 1$, $a = 1$ and $c = 0$. Now part of complementary function (C.F.) corresponding to the factor $(D - D')$ is

 $e^{(0.x/(1))}f_1(1.y+1.x) \implies f_1(y+x).$

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Hence the required solution is

 $z = f_1(y+x) + e^{-x} f_2(y-x),$

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Solve the PDE $r + 2s + t + 2p + 2q + z = 0$.

Solution: The given PDE can be written as $(D^2 + 2DD' + D'2 + 2D + 2D' + 1)z = 0$, which is reducible because it can be written as a linear factor

> $[(D+D')^2+2D+2D'+1]z=0 \implies$ $[(D+D')^2+2(D+D')+1]z=0.$

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e^{((-1).x/(1))}\left\{f_1(1.y-1.x)+xf_2(1.y-1.x)\right\} \implies
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Hence the required solution is

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By comparing $(D + D' + 1)$ with $(bD - aD' - c)$, we get $b = 1$, $a = -1$ and $c = -1$. Now part of complementary function (C.F.) corresponding to the factor $(D+D^\prime+1)^2$ is

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e^{((-1).x/(1))}\left\{f_1(1.y-1.x)+xf_2(1.y-1.x)\right\} \implies e^{-x}\left\{f_1(y-x)+xf_2(y-x)\right\}.
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Example

Solve the PDE $(3D - 5)(7D' + 2)DD'(2D + 3D' + 5)$.

Solution: The given PDE is in a linear factor. Hence the required solution is

 $z = e^{5x/3} f_1(3y) + e^{-(2y/7)} f_2(7x) + f_3(y) + f_4(x) +$ $e^{(-5x/2)}f_5(2y-3x),$

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Exercise

Solve the following PDE:

(1)
$$
(D^2 - DD' + D' - 1)z = 0
$$
 Ans.
\n $z = e^x f_1(y) + e^{-x} f_2(y + x)$
\n(2) $s + p - q - z = 0$ Ans. $e^x f_1(y) + e^{-y} f_2(x)$
\n(3) $(D^2 - DD' - 2D'^2 + 2D + 2D')z = 0$ Ans.
\n $z = f_1(y - x) + e^{-2x} f_2(y + 2x)$
\n(4) $(D^2 - D'^2 + D - D')z = 0$ Ans.
\n $z = f_1(y + x) + e^{-x} f_2(y - x)$
\n(5) $(DD' + aD + bD' + ab)z = 0$ Ans.
\n $z = e^{-bx} f_1(y) + e^{-ay} f_2(x)$

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