# Mathematics-II (Differential Equations)
## Lecture Notes
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by

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# Non-Homogeneous Linear Partial Differential Equations with Constants Coefficients

## Definition

A linear partial differential equations with constant coefficients, which are not homogeneous are called Non-homogeneous.

## Example

$$2\frac{\partial^3 z}{\partial z^3} - 3\frac{\partial^2 z}{\partial z^2} + \frac{\partial z}{\partial z} + 2z = x + 2y$$

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$$\frac{\partial^3 z}{\partial z^3} + \frac{\partial z}{\partial z} - 4z = \sin(x + 2y)$$

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A linear differential operator $F(D, D')$ is known as **reducible**, if it can be written as the product of linear factors of the form $aD + bD' + c$, where $a$, $b$ and $c$ are constants. $F(D, D')$ is known as **irreducible**, if it is not reducible.

## Example

$D^2 - D'^2$ is reducible because it can be written as a linear factor $(D^2 - D'^2) = (D - D')(D + D')$

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**Example**

$D^2 - D'^3$ is irreducible because it can not be written as a linear factor.

## Working rule for finding C.F. of reducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given reducible non-homogeneous linear partial differential equations with constants coefficients be
$F(D, D')z = \phi(x, y)$
**Step-I**: Factorize $F(D, D')$ into linear factors.
**Step-II**: Corresponding to each non-repeated factor $(bD - aD' - c)$, the part of complementary function is taken as $e^{(cx/b)} f_1(by + ax)$, if $b \neq 0$.
**Step-III**: Corresponding to repeated factor $(bD - aD' - c)^r$, the part of complementary function is taken as
$e^{(cx/b)} \left[ f_1(by + ax) + x f_2(by + ax) + x^2 f_3(by + ax) + ... + x^{(r-1)} f_r(by + ax) \right]$, if $b \neq 0$.

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**Step-IV**: Corresponding to each non-repeated factor $(bD - aD' - c)$, the part of complementary function is taken as $e^{-(cy/a)} f_{-1}(by + ax)$, if $a \neq 0$.

**Step-V**: Corresponding to repeated factor $(bD - aD' - c)^r$, the part of complementary function is taken as $e^{-(cy/a)} \left[ f_1(by + ax) + y f_2(by + ax) + y^2 f_3(by + ax) + ... + y^{(r-1)} f_r(by + ax) \right]$, if $a \neq 0$, $f_1, f_2, f_3, ..., f_r$ are arbitrary functions.

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**Example**

Solve the PDE $(D^2 - D'^2 + D - D')z = 0$.

**Solution:** The given PDE $(D^2 - D'^2 + D - D')z = 0$ is reducible because it can be written as a linear factor

$$[(D - D')(D + D') + D - D']z = 0 \implies (D - D')(D + D' + 1)z = 0.$$

By comparing $(D - D')$ with $(bD - aD' - c)$, we get $b = 1$, $a = 1$ and $c = 0$.Now part of complementary function (C.F.) corresponding to the factor $(D - D')$ is

$$e^{(0.x/(1))} f_1(1.y + 1.x) \implies f_1(y + x).$$

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Again by comparing $(D + D' + 1)$ with $(bD - aD' - c)$, we get $b = 1$, $a = -1$ and $c = -1$. Now part of complementary function (C.F.) corresponding to the factor $(D + D' + 1)$ is

$$e^{((-1).x/(1))}\phi(1.y + (-1).x) \implies e^{-x} f_2(y - x).$$

Hence the required solution is

$$z = f_1(y + x) + e^{-x} f_2(y - x),$$

where $f_1$ and $f_2$ are arbitrary function.

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Solve the PDE $r + 2s + t + 2p + 2q + z = 0.$

**Solution:** The given PDE can be written as $(D^2 + 2DD' + D'2 + 2D + 2D' + 1)z = 0$,which is reducible because it can be written as a linear factor

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Solve the PDE $(3D-5)(7D'+2)DD'(2D+3D'+5)$.

**Solution:** The given PDE is in a linear factor.Hence the required solution is

$$z = e^{5x/3}f_1(3y) + e^{-(2y/7)}f_2(7x) + f_3(y) + f_4(x) + e^{(-5x/2)}f_5(2y-3x),$$

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## Exercise

Solve the following PDE:

(1) $(D^2 - DD' + D' - 1)z = 0$ **Ans.** $z = e^x f_1(y) + e^{-x} f_2(y + x)$

(2) $s + p - q - z = 0$ **Ans.** $e^x f_1(y) + e^{-y} f_2(x)$

(3) $(D^2 - DD' - 2D'^2 + 2D + 2D')z = 0$ **Ans.** $z = f_1(y - x) + e^{-2x} f_2(y + 2x)$

(4) $(D^2 - D'^2 + D - D')z = 0$ **Ans.** $z = f_1(y + x) + e^{-x} f_2(y - x)$

(5) $(DD' + aD + bD' + ab)z = 0$ **Ans.** $z = e^{-bx} f_1(y) + e^{-ay} f_2(x)$

Thanks !!!