

Lok Nayak Jai Prakash Institute of Technology Chapra, Bihar-841302

Non-Homogeneous Linear Partial Differential ...

> Dr. G.K. Prajapati

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Introduction

Mathematics-II (Differential Equations) Lecture Notes April 8, 2020

by

Dr. G.K.Prajapati Department of Applied Science and Humanities LNJPIT, Chapra, Bihar-841302

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Non-Homogeneous Linear Partial Differential Equations with Constants Coefficients

Definition

A linear partial differential equations with constant coefficients, which are not homogeneous are called Non-homogeneous.

Example

$$2\frac{\partial^3 z}{\partial z^3} - 3\frac{\partial^2 z}{\partial z^2} + \frac{\partial z}{\partial z} + 2z = x + 2y$$

Example

$$\frac{\partial^3 z}{\partial z^3} + \frac{\partial z}{\partial z} - 4z = \sin(x + 2y)$$

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Definition

A linear differential operator F(D, D') is known as **reducible**, if it can be written as the product of linear factors of the form aD + bD' + c, where a, b and c are constants.F(D, D') is known as **irreducible**, if it is not reducible.

Example

 $D^2-D^{\prime 2}$ is reducible because it can be written as a linear factor $(D^2-D^{\prime 2})=(D-D^\prime)(D+D^\prime)$

Example

 $D^3D'-DD'^3$ is reducible because it can be written as a linear factor $D^3D'-DD'^3=DD'(D-D')(D+D')$

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Introduction	$D^2-D^{\prime 3}$ is irreducible because it can not be written as a linear
	factor.

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Working rule for finding C.F. of reducible non-homogeneous linear partial differential equations with constants coefficients.

Step-I: Factorize F(D, D') into linear factors. イロト イポト イヨト イヨト



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Working rule for finding C.F. of reducible non-homogeneous linear partial differential equations with constants coefficients.

Let the given reducible non-homogeneous linear partial differential equations with constants coefficients be $F(D, D')z = \phi(x, y)$

Step-I: Factorize F(D, D') into linear factors. **Step-II**: Corresponding to each non-repeated factor (bD - aD' - c), the part of complementary function is taken as $e^{(cx/b)}f_1(by + ax)$, if $b \neq 0$.

Step-III: Corresponding to repeated factor $(bD - aD' - c)^r$, the part of complementary function is taken as $e^{(cx/b)} \left[f_1(by + ax) + xf_2(by + ax) + x^2f_3(by + ax) + ... + x^{(r-1)}f_r(by + ax) \right]$, if $b \neq 0$.



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Step-IV: Corresponding to each non-repeated factor (bD - aD' - c), the part of complementary function is taken as $e^{-(cy/a)}f - (by + ax)$, if $a \neq 0$.

Step-V: Corresponding to repeated factor $(bD - aD' - c)^r$, the part of complementary function is taken as $e^{-(cy/a)} \left[f_1(by + ax) + yf_2(by + ax) + y^2f_3(by + ax) + ... + y^{(r-1)}f_r(by + ax) \right]$, if $a \neq 0, f_1, f_2, f_3, ..., f_r$ are arbitrary functions.

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Example

Solve the PDE $(D^2 - D'^2 + D - D')z = 0.$

Solution: The given PDE $(D^2 - D'^2 + D - D')z = 0$ is reducible because it can be written as a linear factor

$$[(D - D')(D + D') + D - D']z = 0 \implies (D - D')(D + D' + 1)z = 0.$$

By comparing (D - D') with (bD - aD' - c), we get b = 1, a = 1 and c = 0.Now part of complementary function (C.F.) corresponding to the factor (D - D') is

 $e^{(0.x/(1))}f_1(1.y+1.x) \implies f_1(y+x).$

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Again by comparing (D + D' + 1) with (bD - aD' - c), we get b = 1, a = -1 and c = -1. Now part of complementary function (C.F.) corresponding to the factor (D + D' + 1) is $\binom{(-1)}{2} \frac{r}{2} \binom{(1)}{2} \binom{(1-1)}{2} \frac{r}{2} \binom{(1-1)}{2} \frac{r}{2} \binom{(1-1)}{2} \binom{($

 $e^{((-1).x/(1))}\phi(1.y+(-1).x) \implies e^{-x}f_2(y-x).$

Hence the required solution is

 $z = f_1(y+x) + e^{-x}f_2(y-x),$

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Example

Solve the PDE r + 2s + t + 2p + 2q + z = 0.

Solution: The given PDE can be written as $(D^2 + 2DD' + D'2 + 2D + 2D' + 1)z = 0$, which is reducible because it can be written as a linear factor

 $[(D+D')^2 + 2D + 2D' + 1]z = 0 \implies [(D+D')^2 + 2(D+D') + 1]z = 0.$

 $[(D+D'+1)^2]z = 0.$

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Solution: The given PDE can be written as $(D^2 + 2DD' + D'2 + 2D + 2D' + 1)z = 0$,which is reducible because it can be written as a linear factor

$$\begin{split} [(D+D')^2 + 2D + 2D' + 1]z &= 0 \implies \\ [(D+D')^2 + 2(D+D') + 1]z &= 0. \\ [(D+D'+1)^2]z &= 0. \end{split}$$

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By comparing (D + D' + 1) with (bD - aD' - c), we get b = 1, a = -1 and c = -1. Now part of complementary function (C.F.) corresponding to the factor $(D + D' + 1)^2$ is

$$e^{((-1).x/(1))} \{ f_1(1.y-1.x) + x f_2(1.y-1.x) \} \implies e^{-x} \{ f_1(y-x) + x f_2(y-x) \}.$$

Hence the required solution is

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$$e^{2((-1).x/(1))} \{ f_1(1.y-1.x) + x f_2(1.y-1.x) \} \implies e^{-x} \{ f_1(y-x) + x f_2(y-x) \}.$$

Hence the required solution is

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$$z = e^{-x} \{ f_1(y - x) + x f_2(y - x) \},\$$

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Example

Solve the PDE $(3D-5)(7D^\prime+2)DD^\prime(2D+3D^\prime+5).$

Solution: The given PDE is in a linear factor.Hence the required solution is

 $\begin{aligned} z &= e^{5x/3} f_1(3y) + e^{-(2y/7)} f_2(7x) + f_3(y) + f_4(x) + \\ &e^{(-5x/2)} f_5(2y-3x), \end{aligned}$

where f_1 , f_2 , f_3 , f_4 and f_5 are arbitrary function.

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Solve the PDE $(3D-5)(7D^\prime+2)DD^\prime(2D+3D^\prime+5).$

Solution: The given PDE is in a linear factor.Hence the required solution is

 $z = e^{5x/3} f_1(3y) + e^{-(2y/7)} f_2(7x) + f_3(y) + f_4(x) + e^{(-5x/2)} f_5(2y - 3x),$

where f_1 , f_2 , f_3 , f_4 and f_5 are arbitrary function.

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Example

Solve the PDE (3D - 5)(7D' + 2)DD'(2D + 3D' + 5).

Solution: The given PDE is in a linear factor. Hence the required solution is

 $\begin{aligned} z &= e^{5x/3} f_1(3y) + e^{-(2y/7)} f_2(7x) + f_3(y) + f_4(x) + \\ &e^{(-5x/2)} f_5(2y-3x), \end{aligned}$

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Example

Solve the PDE $(3D-5)(7D^\prime+2)DD^\prime(2D+3D^\prime+5).$

Solution: The given PDE is in a linear factor. Hence the required solution is

$$\begin{split} z &= e^{5x/3} f_1(3y) + e^{-(2y/7)} f_2(7x) + f_3(y) + f_4(x) + \\ &e^{(-5x/2)} f_5(2y-3x), \end{split}$$

where f_1 , f_2 , f_3 , f_4 and f_5 are arbitrary function.

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Exercise

Solve the following PDE:

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(1)
$$(D^2 - DD' + D' - 1)z = 0$$
 Ans.
 $z = e^x f_1(y) + e^{-x} f_2(y + x)$
(2) $s + p - q - z = 0$ Ans. $e^x f_1(y) + e^{-y} f_2(x)$
(3) $(D^2 - DD' - 2D'^2 + 2D + 2D')z = 0$ Ans.
 $z = f_1(y - x) + e^{-2x} f_2(y + 2x)$
(4) $(D^2 - D'^2 + D - D')z = 0$ Ans.
 $z = f_1(y + x) + e^{-x} f_2(y - x)$
(5) $(DD' + aD + bD' + ab)z = 0$ Ans.
 $z = e^{-bx} f_1(y) + e^{-ay} f_2(x)$

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Thanks !!!

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