Mathematics-II (Differential Equations)
Lecture Notes
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by

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Introduction

Short Method to find the Particular Integral

Short Method-II (When right hand side function is of the form $\phi(x^m y^n)$ i.e. $F(D, D') = \phi(x^m y^n)$), where m and n are either integer or rational number.

Let $F(D, D') = \phi(x^m y^n)$ be homogeneous function of D and D' of order n. Then the particular integral is defined as

$$\frac{1}{F(D, D')} \phi(x^m y^n),$$

Then particular integral evaluated by expanding the symbolic function $\dfrac{1}{F(D, D')}$ in an infinite series of ascending power of D or D'.

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Remark-1: If $n \leq m$, then $\dfrac{1}{F(D, D')}$ should be expanded in powers of $\dfrac{D'}{D}$ whereas If $m \leq n$, then $\dfrac{1}{F(D, D')}$ should be expanded in powers of $\dfrac{D}{D'}$.

Remark-2: Binomial expansion

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Example

Solve $(D^2 - a^2D'^2)z = x$.

Solution: The auxiliary equation is $m^2 - a^2 = 0$, which gives $m = -a, +a$. Therefore it's complementary function (C.F.) is

$C.F. = f_1(y - ax) + f_2(y + ax)$, where f_1, f_2 are arbitrary function.

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Now, Particular Integral (P.I.) will be

$$P.I. = \frac{1}{F(D,D')}\phi(x^m y^n) = \frac{1}{D^2 - a^2 D'^2}(x)$$

$$= \frac{1}{D^2\left[1 - \left(\frac{a^2 D'^2}{D^2}\right)\right]}(x)$$

$$= \frac{1}{D^2}\left[1 - \left(\frac{a^2 D'^2}{D^2}\right)\right]^{-1}(x)$$

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$$P.I. = y^3\frac{x^6}{120} + \frac{x^9}{10080}.$$

Therefore the required general solution is $z = C.F. + P.I.$i.e.

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$$P.I. = y^3\frac{x^6}{120} + \frac{x^9}{10080}.$$

Therefore the required general solution is $z = C.F. + P.I.$i.e.

$$z = f_1(y+x) + f_2(y+\omega x) + f_3(y+\omega^2 x) + \frac{x^6y^3}{120} + \frac{x^9}{10080}.$$

Solve the following PDE:

(1) $(D^2 - 6DD' + 9D'^2)z = 12x^2 + 36xy$ **Ans.** $z = f_1(y+3x) + xf_2(y+3x) + 10x^4 + 6x^3y$

(2) $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$ **Ans.** $z = f_1(y+x) + xf_2(y+x) + e^(x+2y) + \frac{x^5}{20}$

(3) $(D^3 - 7DD'^2 - 6D'^3)z = x^2 + xy^2 + y^3$

Thanks !!!