Mathematics-II (Differential Equations)
Lecture Notes
April 7, 2020

by

Dr. G.K.Prajapati
Department of Applied Science and Humanities
LNJPIT, Chapra, Bihar-841302

Short Method to find the Particular Integral

Short Method-III (General Method).
Let $F(D, D') = \phi(x, y)$ be homogeneous function of D and D' of order n. The particular integral is defined as

$$\frac{1}{F(D, D')}\phi(x, y),$$

Let the particular integral can be written as

$$\frac{1}{(D - m_1D')(D - m_2D')(D - m_3D')...(D - m_nD')}\phi(x, y),$$

Short Method to find the Particular Integral

Short Method-III (General Method).
Let $F(D, D') = \phi(x, y)$ be homogeneous function of D and D' of order n. The particular integral is defined as

$$\frac{1}{F(D, D')}\phi(x, y),$$

Let the particular integral can be written as

$$\frac{1}{(D - m_1 D')(D - m_2 D')(D - m_3 D')...(D - m_n D')}\phi(x, y),$$

The we use one of the following formula

$$\frac{1}{(D - m_1 D')}\phi(x, y) = \int \phi(x, c - mx)dx,$$
where $c = y + mx$.

or

$$\frac{1}{(D + m_1 D')}\phi(x, y) = \int \phi(x, c - mx)dx,$$
where $c = y - mx$.

Example

Solve $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = \sin x.$

Solution: The auxiliary equation is $m + 1 = 0$, which gives $m = -1$. Therefore it's complementary function (C.F.) is

$$C.F. = f_1(y - x), \text{ where } f_1 \text{ is arbitrary function.}$$

Now, Particular Integral (P.I.) will be

$$P.I. = \frac{1}{F(D,D')}\phi(x,y) = \frac{1}{(D+D')}\sin x$$
$$= \int \{\sin x\}\, dx,$$
$$P.I. = -\cos x$$

Therefore the required general solution is $z = C.F. + P.I.$i.e.

$$z = f_1(y-x) - \cos x$$

Example

Solve $(D^2 - DD' - 2D'^2)z = (y-1)e^x$.

Solution: The auxiliary equation is $m^2 - m - 2 = 0$, which gives $m = -1, 2$. Therefore it's complementary function (C.F.) is

$$C.F. = f_1(y-x) + f_2(y+2x), \text{ where } f_1, f_2 \text{ are arbitrary function.}$$

Now, Particular Integral (P.I.) will be

$$P.I. = \frac{1}{F(D,D')}\phi(x,y) = \frac{1}{(D+D')(D-2D')}(y-1)e^{x}$$

$$= \frac{1}{(D+D')}\left\{\frac{1}{(D-2D')}(y-1)e^{x}\right\}$$

$$= \frac{1}{(D+D')}\int\{(c-2x-1)e^{x}\}\,dx,$$

$$\therefore \qquad c = y+2x$$

$$= \frac{1}{(D+D')}\left\{(c-2x-1)e^{x} - \int(-2)e^{x}dx\right\}$$

$$= \frac{1}{(D+D')}\{(c-2x-1)e^x + 2e^x\}$$

$$= \frac{1}{(D+D')}\{(c-2x+1)e^x\}$$

$$= \frac{1}{(D+D')}\{(y+2x-2x+1)e^x\}$$

$$\therefore c = y+2x$$

$$= \frac{1}{(D+D')}\{(y+1)e^x\}$$

$$= \int (c' + x + 1)e^x dx$$

$$\therefore c' = y - x$$

$$P.I. = (c' + x + 1)e^x - \int (1.e^x) dx$$

$$= (c' + x + 1)e^x - e^x = ye^x$$

$$\therefore c' = y - x.$$

Therefore the required general solution is $z = C.F. + P.I.$i.e.

$$z = f_1(y - x) + f_2(y + 2x) + ye^x.$$

Example

Solve $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy$.

Solution: The auxiliary equation is $m^2 - m - 2 = 0$, which gives $m = -1, 2$. Therefore it's complementary function (C.F.) is

$$C.F. = f_1(y - x) + f_2(y + 2x), \text{ where } f_1, f_2 \text{ are arbitrary function.}$$

Now, Particular Integral (P.I.) will be

$$P.I. = \frac{1}{F(D,D')}\phi(x,y)$$

$$= \frac{1}{(D-2D')(D+D')}\left\{(2x^2+xy-y^2)\sin xy - \cos xy\right\}$$

$$= \frac{1}{(D-2D')(D+D')}\left\{(2x-y)(x+y)\sin xy - \cos xy\right\}$$

$$= \frac{1}{(D-2D')}\int \left\{(2x-x-c)(x+x+c)\sin x(x+c) - \cos x(x+c)\right.$$

$$\therefore \quad c = y - x$$

$$=$$

$$\frac{1}{(D-2D')}\int\left\{(x-c)(2x+c)\sin(x^2+cx)-\cos(x^2+cx)\right\}dx$$

$$=$$

$$\frac{1}{(D-2D')}\left\{-(x-c)\cos(x^2+cx)+\int\cos(x^2+cx)dx-\int\cos($$

$$=\frac{1}{(D-2D')}\{(y-2x)\cos xy\}$$

$$\therefore c=y-x$$

$$=\int(c'-2x-2x)\cos x(c'-2x)dx$$

$$= \int (c' - 2x - 2x) \cos x(c' - 2x)dx$$
$$\therefore c' = y + 2x$$
$$= \int (c' - 4x) \cos(xc' - 2x^2)dx$$
$$= Let \quad xc' - 2x^2 = t \quad so \quad that \quad (c' - 4x)dx = dt$$
$$P.I. = \sin(c'x - 2x^2) = \sin xy.$$

Therefore the required general solution is $z = C.F. + P.I.$i.e.

$$z = f_1(y - x) + f_2(y + 2x) + \sin xy.$$

Solve the following PDE:

(1) $(D^2 - 4D'^2)z = \dfrac{4x}{y^2} - \dfrac{y}{x^2}$ **Ans.**
$z = f_1(y+2x) + f_2(y-2x) + x\log y + y\log x + 3x$

(2) $r + s - 6t = y\sin x$ **Ans.**
$z = f_1(y+2x) + f_2(y-3x) - y\sin x - \cos x$

(3) $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$ **Ans.**
$z = f_1(y-x) + xf_2(y-x) + x\sin y$

(4) $r - t = \tan^3 x\tan y - \tan x\tan^3 y$ $[AKU2019]$ **Ans.**
$z = f_1(y-x) + xf_2(y+x) + 1/2\tan y\tan x.$

Thanks !!!